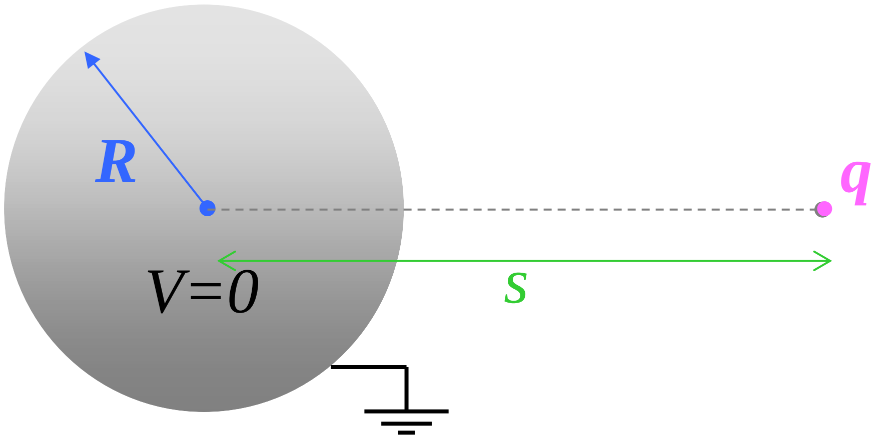
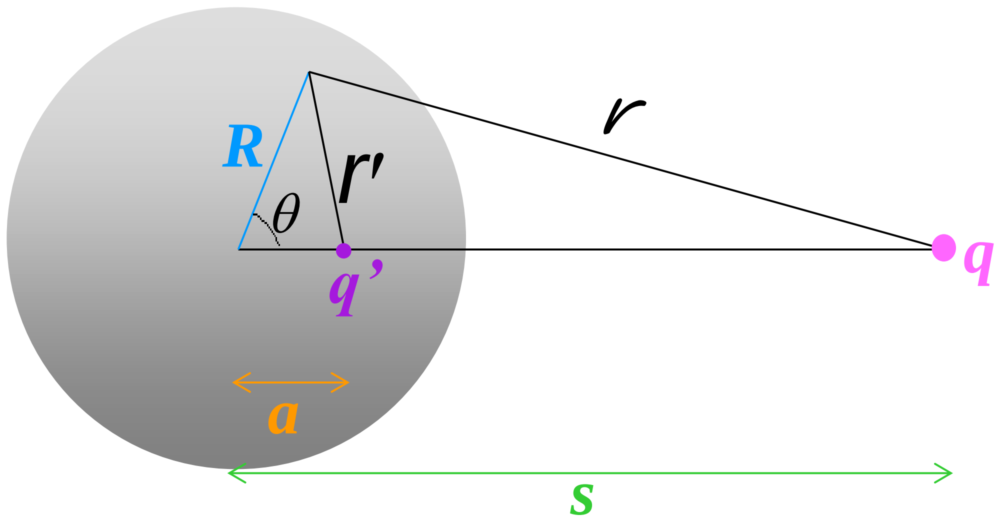
Image Charge Method of a Grounded Conducting Sphere:

Consider a grounded conducting sphere of radius R, with a point charge q located at a distance s from the center and ***outside the sphere, i.e., s > R***. We want to find the potential outside the sphere.

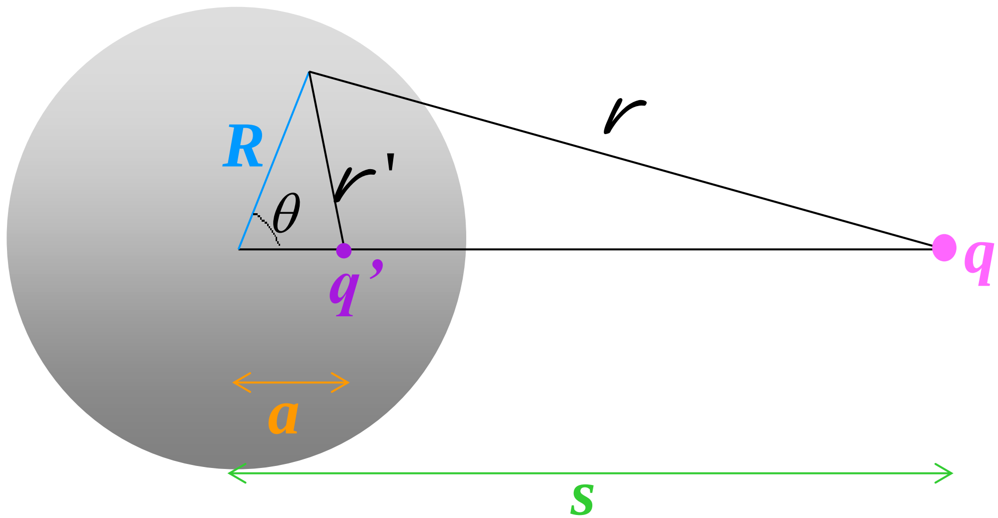


Consider the following system



with an image charge q’ inside at a distance a from the center, i.e., $a < R$.

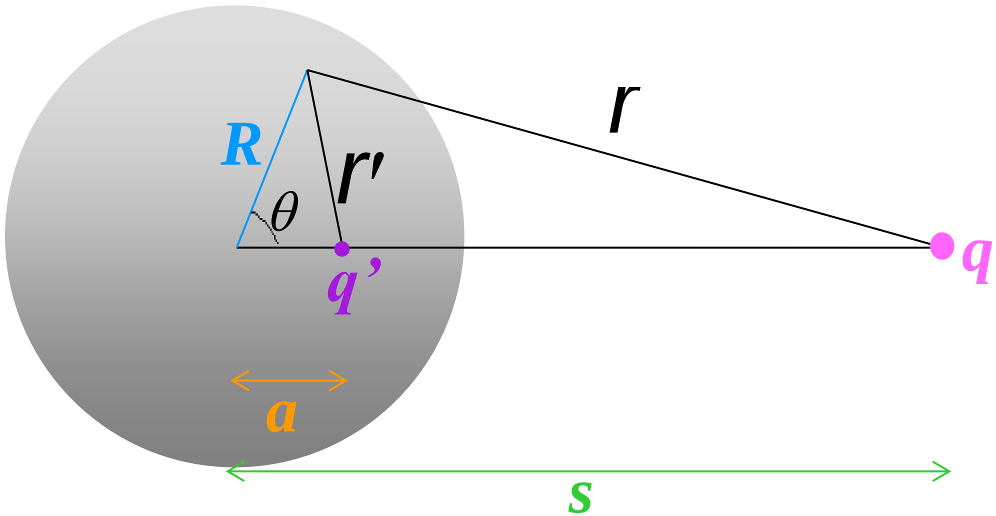
We want to choose *q*’ and *a* such that the potential on the surface of the sphere is 0, i.e.,



$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r}+\frac{q'}{r'}\right]=0$$

$$\Rightarrow \frac{r}{r'}=-\frac{q}{q'}$$

$$\frac{r}{r'} = -\frac{q}{q'}$$



By cosine law,

$$r = \sqrt{R^2 + s^2 - 2Rs\cos\theta}$$

$$r' = \sqrt{R^2 + a^2 - 2Ra\cos\theta}$$

So,

$$\frac{q^2}{q'^2} = \frac{r^2}{r'^2} = \frac{R^2 + s^2 - 2Rs\cos\theta}{R^2 + a^2 - 2Ra\cos\theta}$$

$$q^2\left(R^2 + a^2 - 2Ra\cos\theta\right) = q'^2\left(R^2 + s^2 - 2Rs\cos\theta\right)$$

$$q^2\left(R^2+a^2-2Ra\cos\theta\right)=q'^2\left(R^2+s^2-2Rs\cos\theta\right)$$

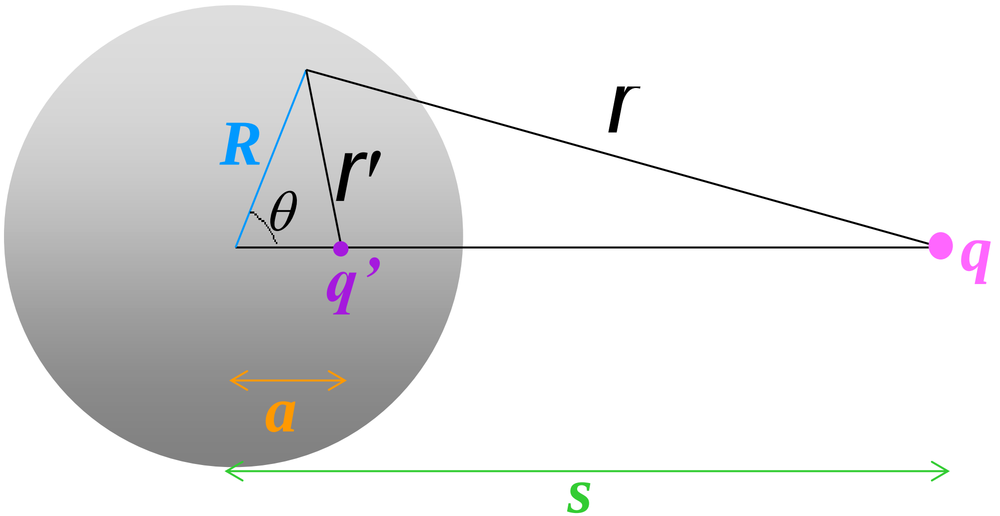
Since this holds for all θ , we have

$$\begin{cases} q^2 a = q'^2 s \\ q^2\left(R^2+a^2\right)=q'^2\left(R^2+s^2\right) \end{cases}$$

$$R^2+a^2=\frac{a}{s}\left(R^2+s^2\right)$$

$$a^2-\frac{R^2+s^2}{s}a+R^2=0$$

$$\Rightarrow a=s \qquad \text{or} \qquad a=\frac{R^2}{s}$$



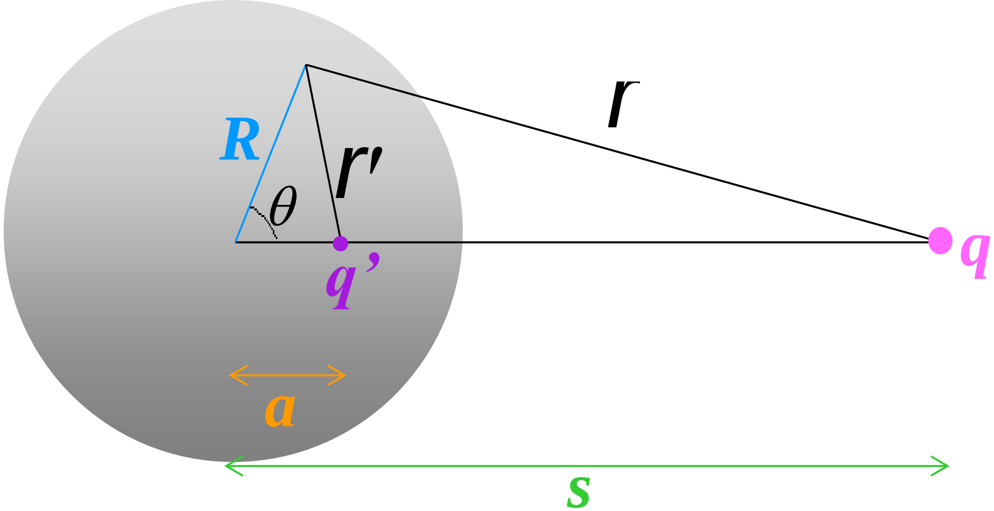
Because $a < R$, the solution $a = s$ is rejected.
Besides,

$$\frac{R^2}{s} < R$$

Therefore, the image charge should be located at

$$\boxed{a = \frac{R^2}{s}}$$

Because $s > R$, so $a < R$
The image charge is inside the sphere

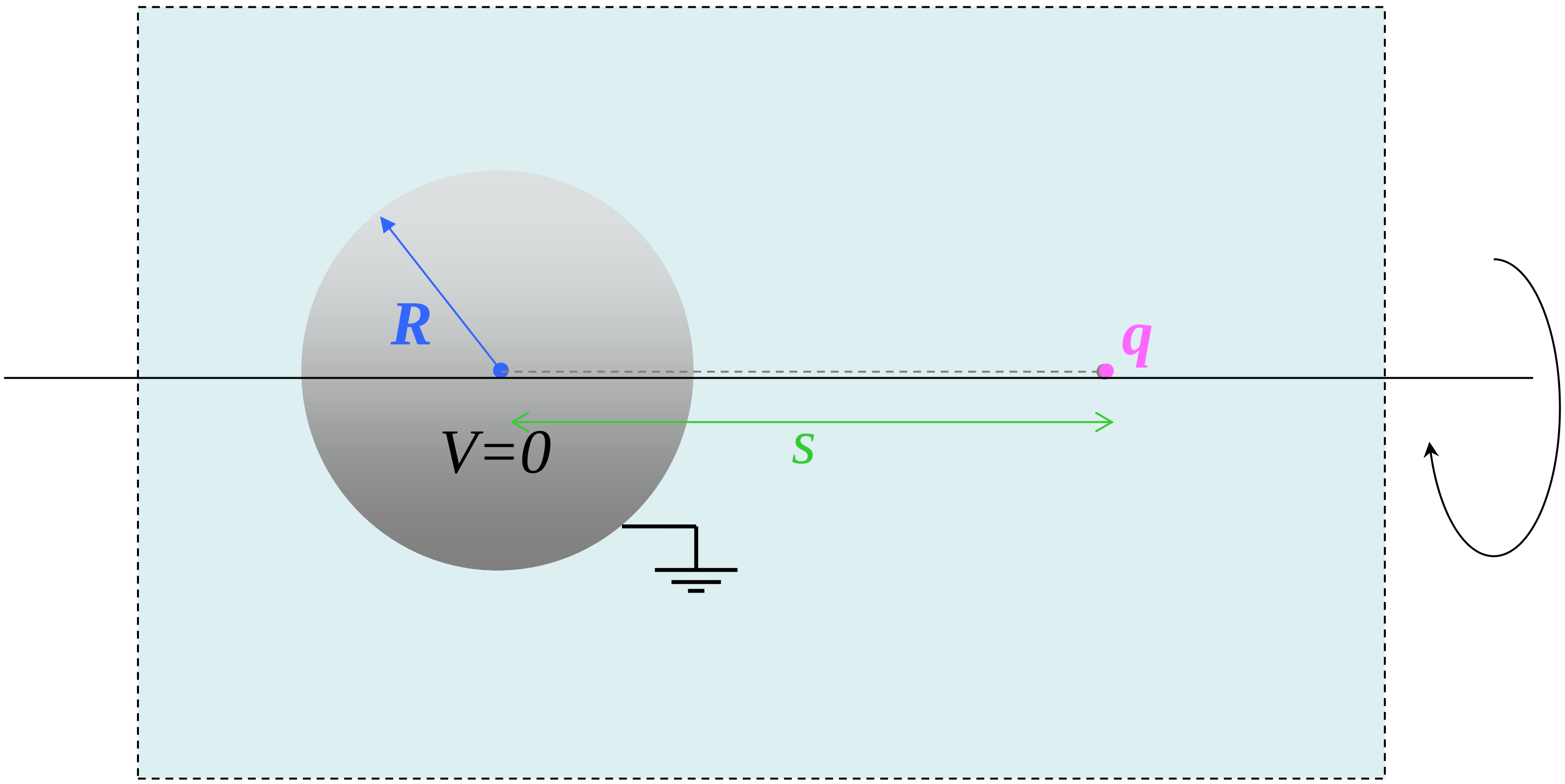


The magnitude of the charge is

$$q' = \pm q\sqrt{\frac{a}{s}} = \pm q\,\frac{R}{s}$$

It is easy to check that the solution with the plus sign is not the solution of the original equation, therefore

$$\boxed{q' = -q\,\frac{R}{s}}$$


R
q
V=0
s