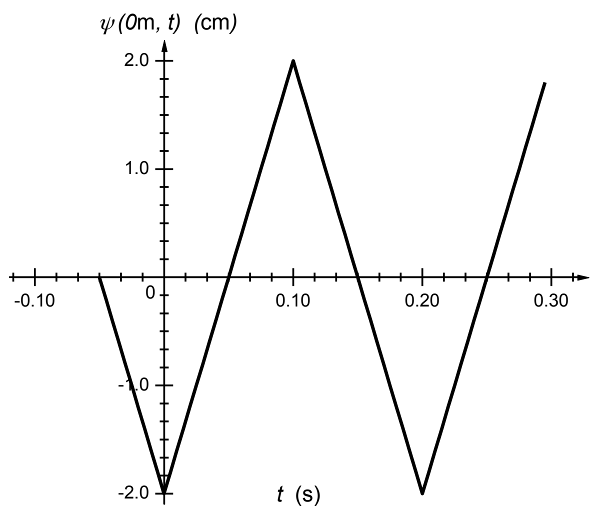
PHYS3038 Fall 2015

Homework #1

Due date: 15 September 2015

2.15 A transverse wave on a string travels in the negative *y*-direction at a speed of 40.0 cm/s. Figure P.2.15 is a graph of ψ versus *t* showing how a point on the rope at *y*=0 oscillates. (a) Determine the wave's period. (b) What is the frequency of the wave? (c) What is the wavelength of the wave? (d) Sketch the profile of the wave (ψ versus *y*).

Figure P.2.15



2.22 Write the expression for the wavefunction of a harmonic wave of amplitude 10^3 V/m, period 2.2×10^{-15} s, and speed 3×10^8 m/s. The wave is propagating in the negative *x*-direction and has a value of 10^3 V/m at *t*=0 and *x*=0.

2.23 Consider the pulse described in terms of its displacement at *t*=0 by

$$y(x,t)|_{t=0} = \frac{C}{2+x^2}$$

where *C* is a constant. Draw the wave profile. Write an expression for the wave, having a speed *v* in the negative *x*-direction, as a function of time t. If *v*=1 m/s, sketch the profile at *t*=2s.

2.32 Determine which of the following describe traveling waves:

$$\text{(a) } \psi(y,t) = e^{-(a^2y^2+b^2t^2-2abty)}$$

$$\text{(b) } \psi(z,t) = A\ sin\ (az^2 - bt^2)$$

$$\text{(c) } \psi(x,t) = A\ sin 2\pi (\frac{x}{a} + \frac{t}{b})^2$$

$$\text{(d) } \psi(x,t) = Acos^2 2\pi(t - x)$$

Where appropriate, draw the profile and find the speed and direction of motion.

2.33 Given the traveling wave $\psi(x,t) = 5.0\ exp(-ax^2 - bt^2 - 2\sqrt{ab}xt)$, determine its direction of propagation. Calculate a few values of ψ and make a sketch of the wave at *t*=0. Taking *a*=25 m^{-2} and *b*=9.0 s^{-2}. What is the speed of the wave?

2.40 Show that Eqs. (2.64) and (2.65), which are plane waves of arbitrary form, satisfy the three-dimensional differential wave equation.

Equation (2.64) $\psi(x,y,z,t) = f(\alpha x + \beta y + \gamma z - vt)$

Equation (2.65) $\psi(x,y,z,t) = g(\alpha x + \beta y + \gamma z + vt)$

3.4 Imagine an electromagnetic wave with its $\vec{\mathbf{E}}$-field in the y-direction. Show that Eq.(3.27)

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

applied to the harmonic wave $\vec{\mathbf{B}}$

$$\vec{\mathbf{E}} = \vec{\boldsymbol{E}}_0 \cos\ (kx - \omega t) \qquad \vec{\mathbf{B}} = \vec{\boldsymbol{B}}_0 \cos\ (kx - \omega t)$$

yields the fact that

$$E_0 = cB_0$$

in agreement with Eq.(3.30): $E_y = cB_z$.

3.5 An electromagnetic wave is specified (in **SI** units) by the following function:

$$\vec{\mathbf{E}} = (-6\hat{\mathbf{i}} + 3\sqrt{5}\hat{\boldsymbol{j}})(10^4 V/m)e^{i[\frac{1}{3}(\sqrt{5}x+2y)\pi\times10^7 - 9.42\times10^{15}t]}$$

Find (a) the direction along which the electric field oscillates, (b) the scalar value of amplitude of the electric field, (c) the direction of propagation of the wave, (d) the propagation number and wavelength, (e) the frequency and angular frequency, and (f) the speed.

3.7 A 550-nm harmonic EM-wave whose electric field is in the *z*-direction is traveling in the *y*-direction in vacuum. (a) What is the frequency of the wave? (b) Determine both ω and *k* for this wave. (c) If the electric field amplitude is 600 V/m, what is the amplitude of the magnetic field? (d) Write an expression for both *E(t)* and *B(t)* given that each is zero at *x*=0 and *t*=0. Put in all the appropriate units.