PHYS 3038 Optics
L9 Propagation of Light
Reading Material: Ch4

Shengwang Du

2015, the Year of Light

Light in Bulk (Dielectric) Matter

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho_f$$

$$\nabla \cdot \vec{B} = 0$$

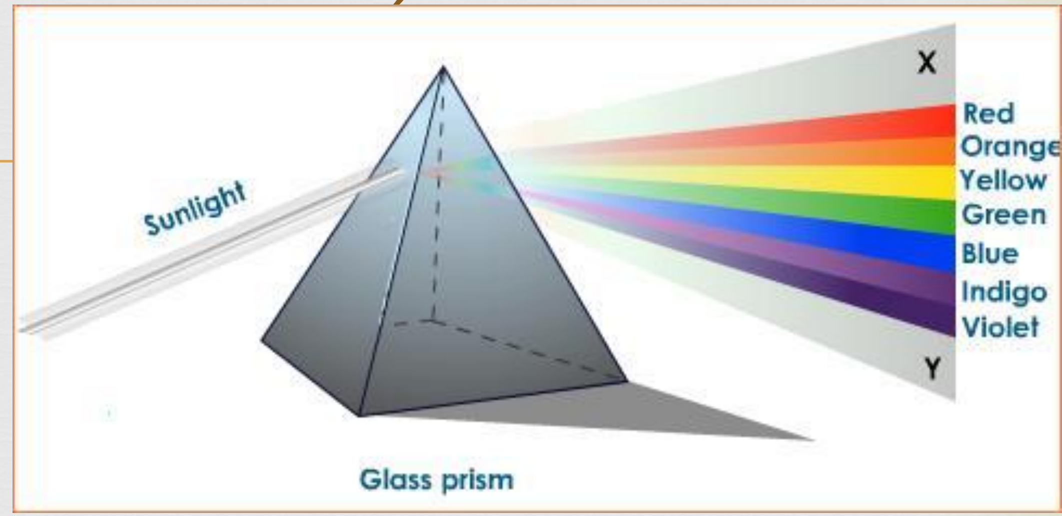
$$\varepsilon_0 \Rightarrow \varepsilon = \varepsilon(\omega)$$

Dispersion

$$\mu_0 \Rightarrow \mu = \mu(\omega)$$

For most material (nonmagnetic) $\mu = \mu(\omega) \cong \mu_0$

Dispersion (Dielectric)



$$v(\omega) = \frac{1}{\sqrt{\varepsilon(\omega)\mu(\omega)}} = \frac{c}{n(\omega)}$$

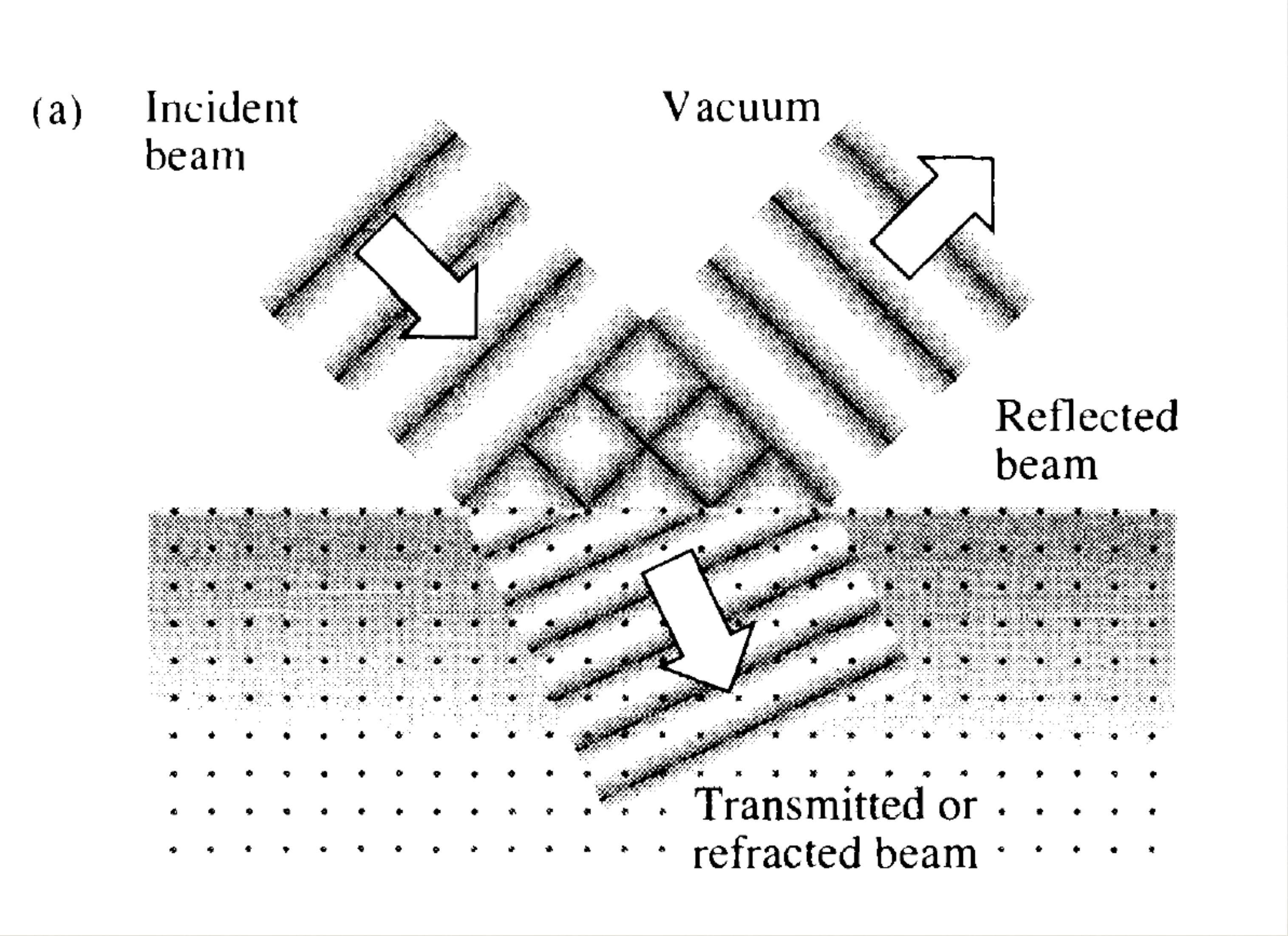
$$n(\omega) = \sqrt{\frac{\varepsilon(\omega)\mu(\omega)}{\varepsilon_0\mu_0}} = \sqrt{K_E(\omega)K_M(\omega)}$$

$$K_E(\omega) = \sqrt{\frac{\varepsilon(\omega)}{\varepsilon_0}}$$

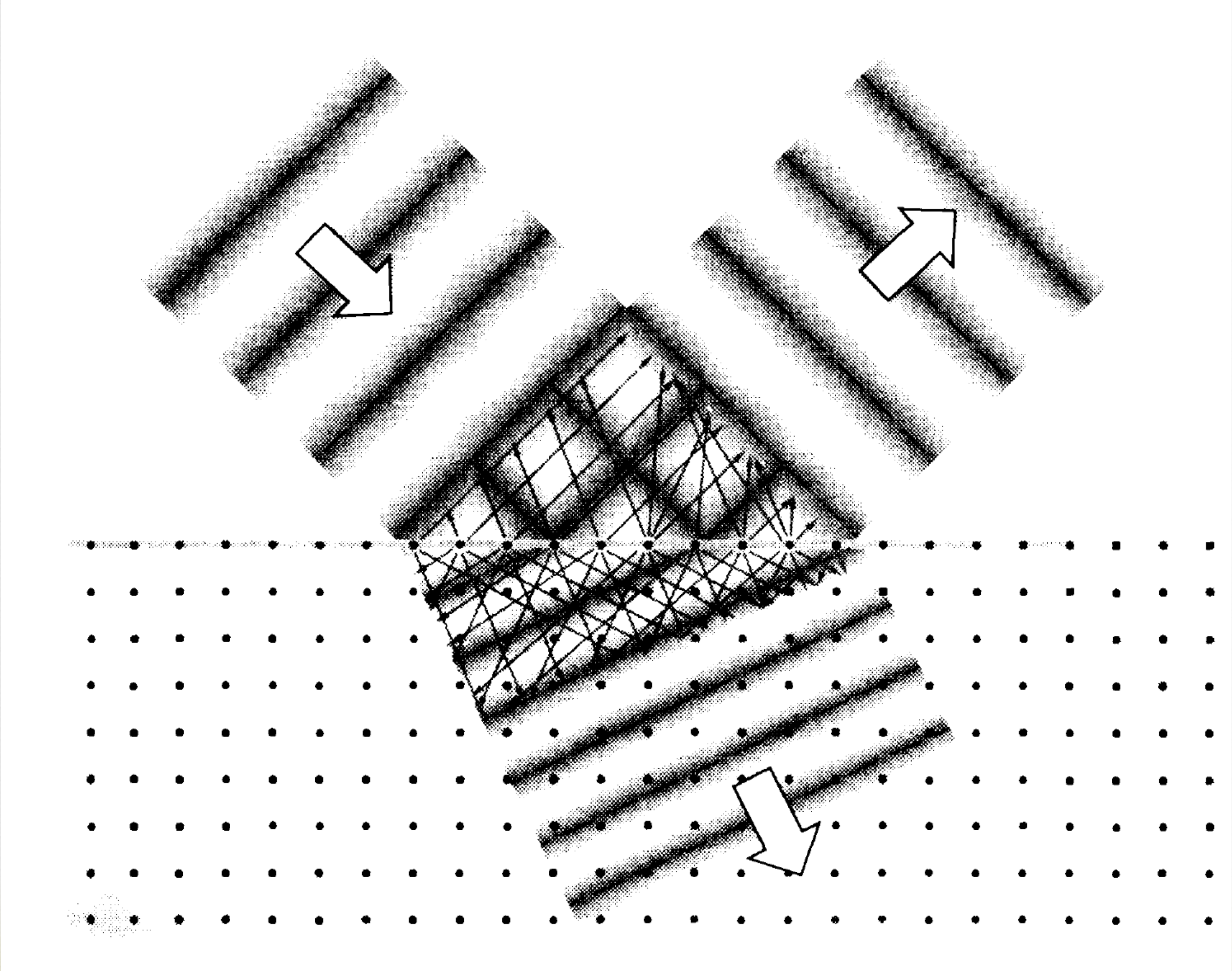
Dielectric constant

$$K_M(\omega) = \sqrt{\frac{\mu(\omega)}{\mu_0}} \approx 1$$

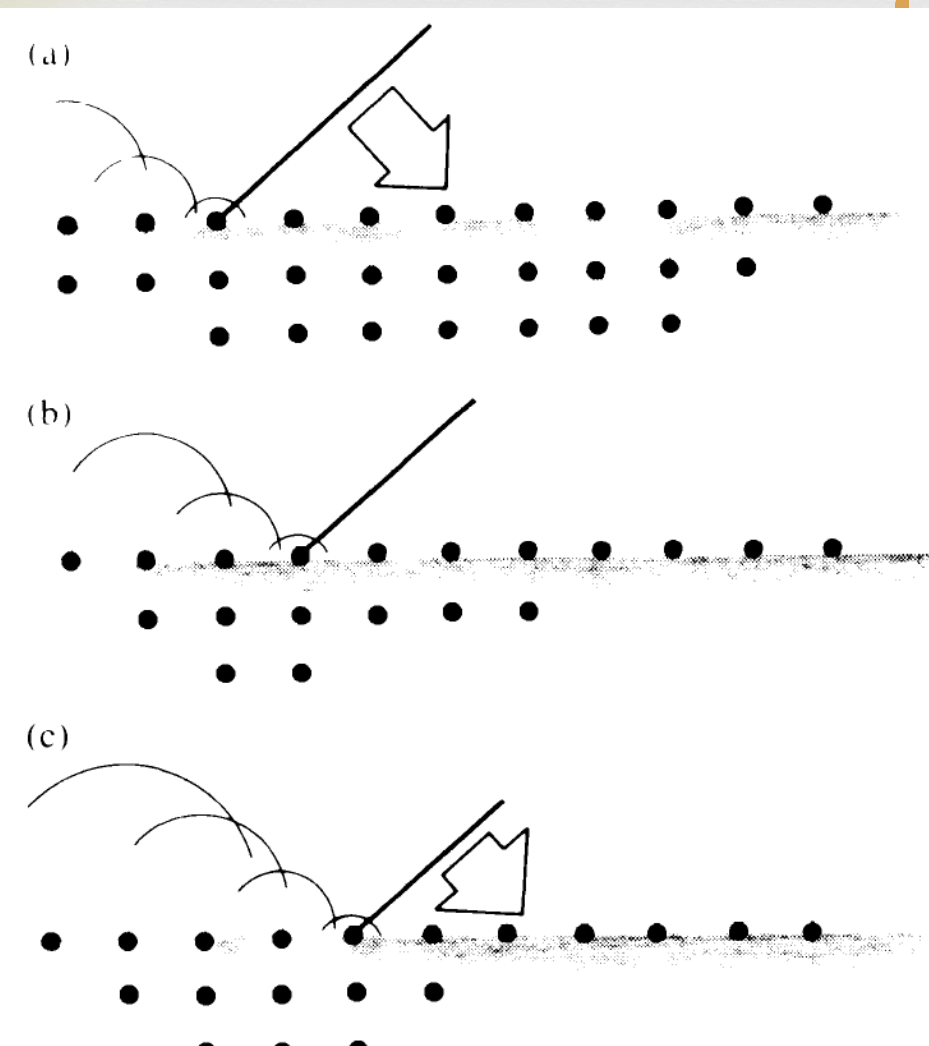
Reflection and Refraction

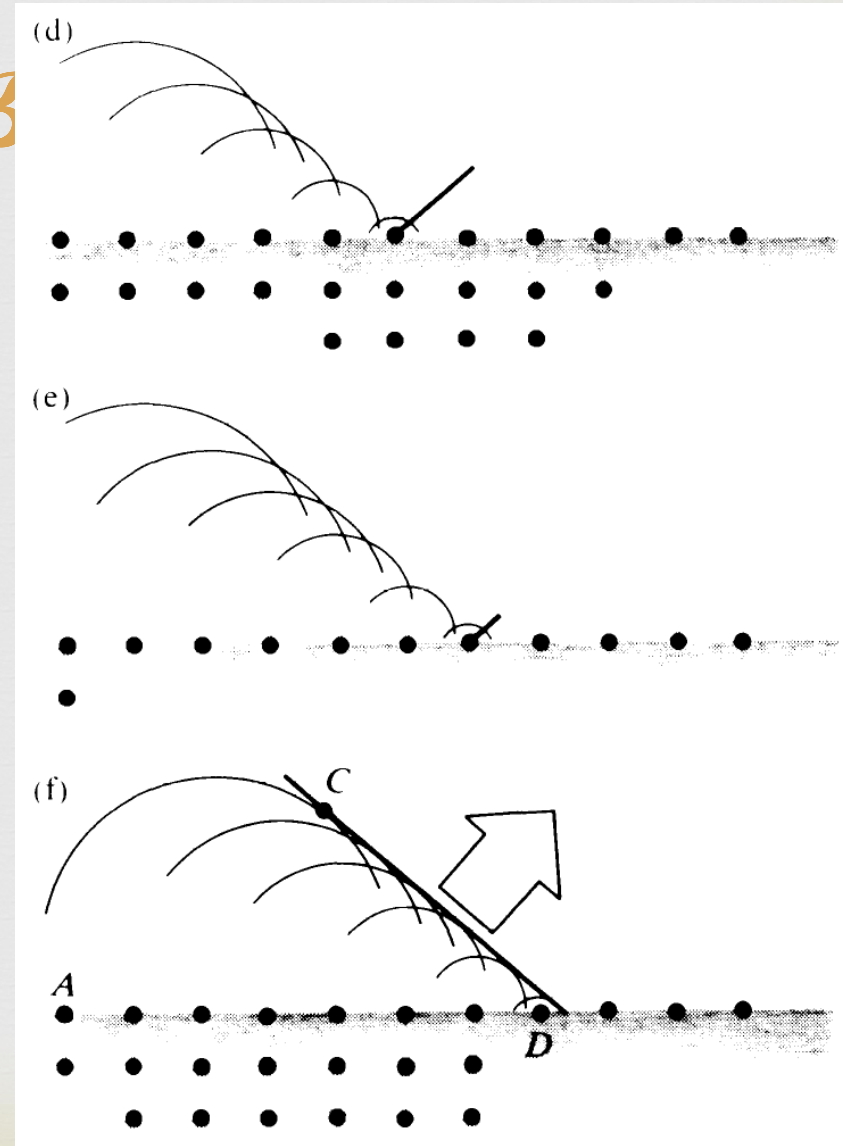


Reflection and Refraction

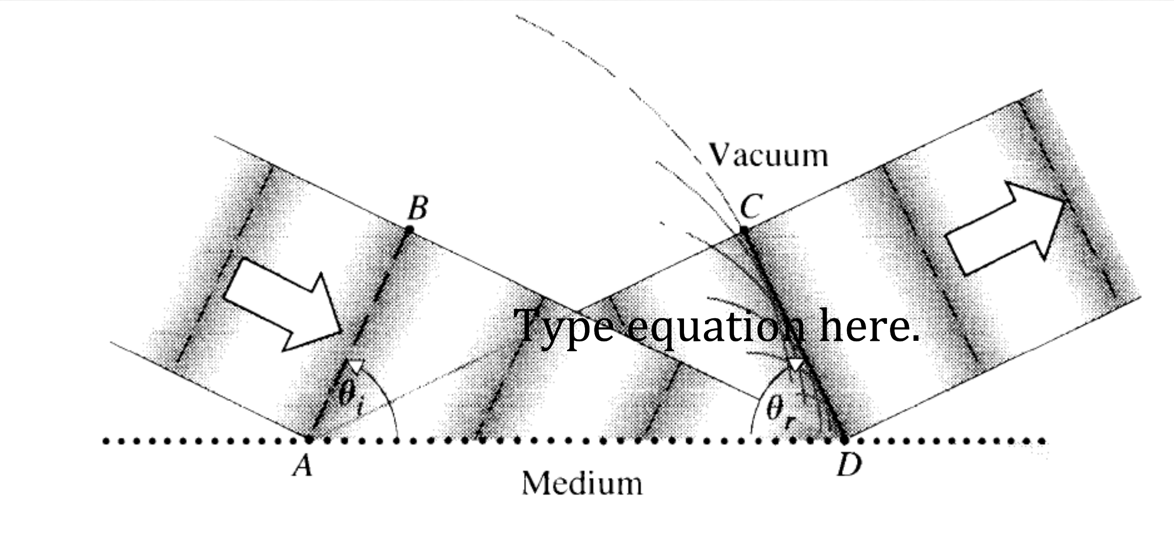


Reflection



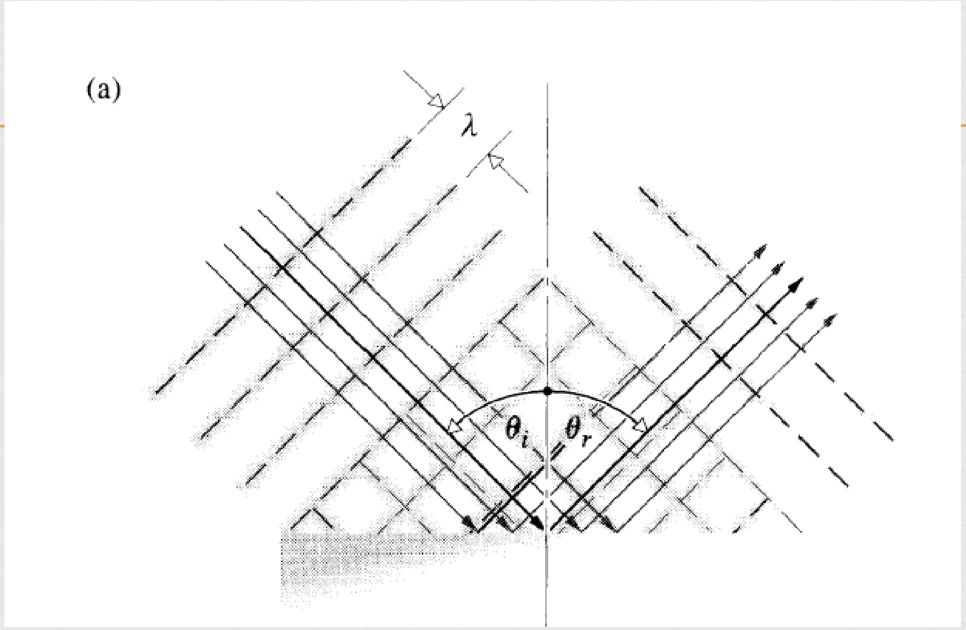


Reflection

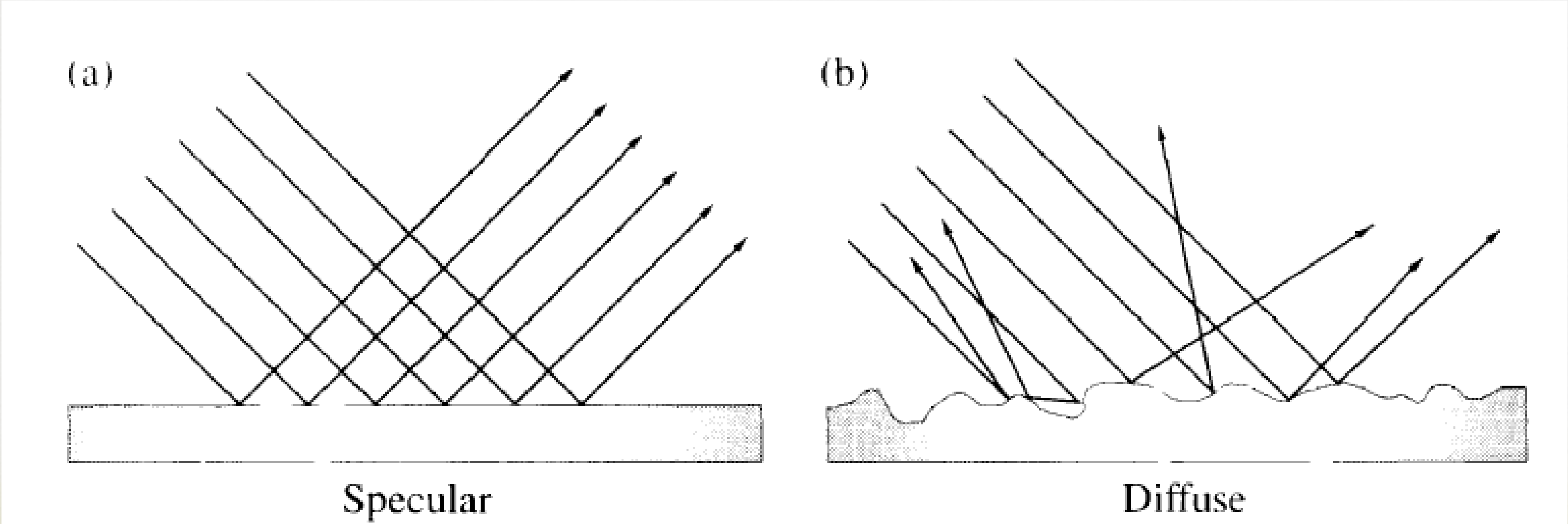


$\overline{AC} = \overline{BD} \longrightarrow \theta_i = \theta_r$

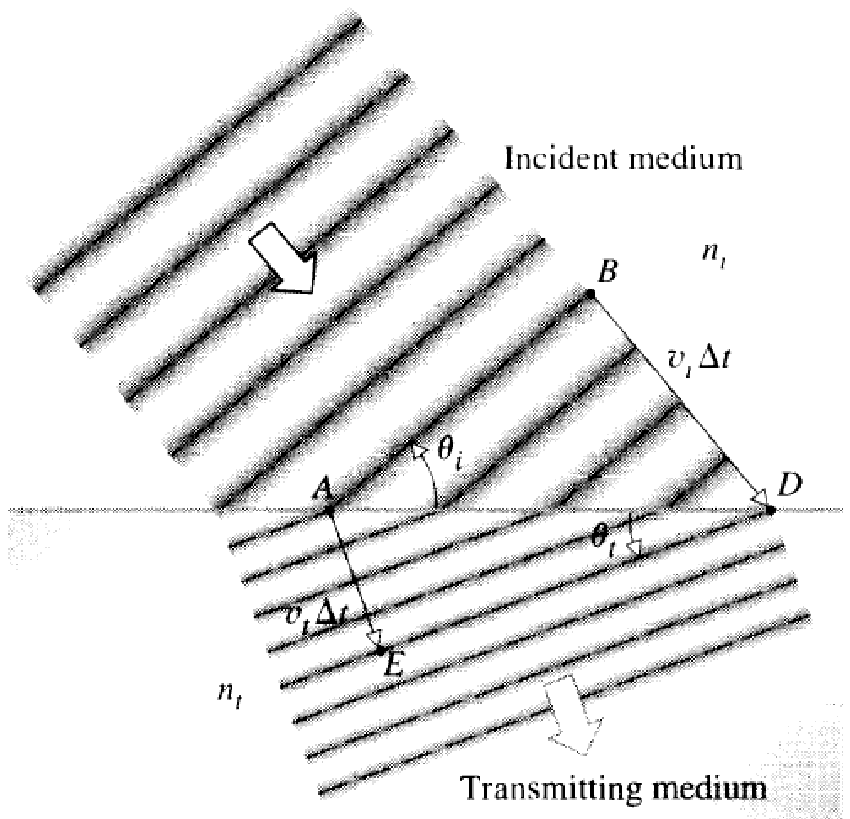
Reflection: Ray



$$\theta_i = \theta_r$$



Refraction

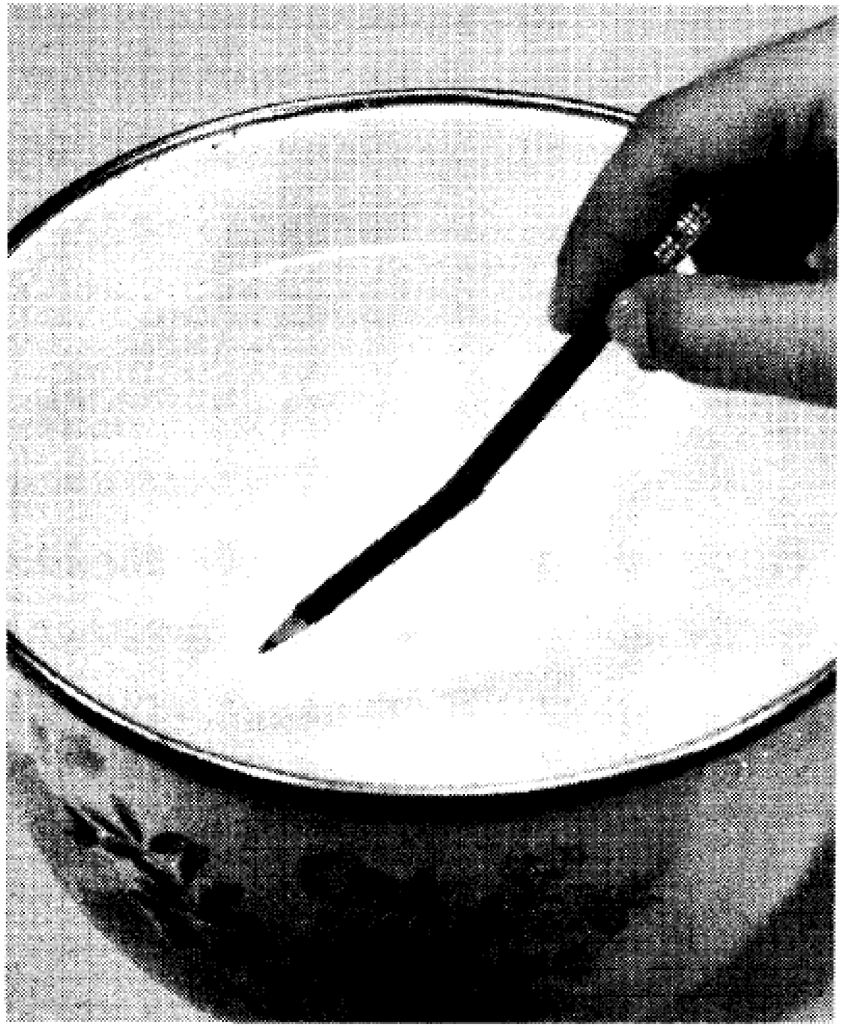


$$n_i \overline{BD} = n_t \overline{AE}$$

$$n_i \overline{AD} \sin\theta_i = n_t \overline{AD} \sin\theta_t$$

$$n_i \sin\theta_i = n_t \sin\theta_t$$

Refraction



Rays from the submerged portion of the pencil bend on leaving the water as they rise toward the viewer. (Photo by E.H.)

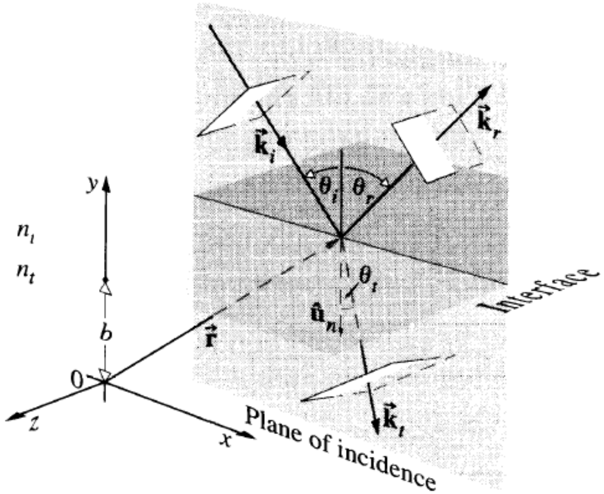


Figure 4.38 Plane waves incident on the boundary between two homogeneous, isotropic, lossless dielectric media.

EM Approach

$$\vec{E}_i = \vec{E}_{i0} \exp[i(\vec{k}_i \cdot \vec{r} - \omega t)]$$

$$\vec{E}_r = \vec{E}_{r0} \exp[i(\vec{k}_r \cdot \vec{r} - \omega t + \varphi_r)]$$

$$\vec{E}_t = \vec{E}_{t0} \exp[i(\vec{k}_t \cdot \vec{r} - \omega t + \varphi_t)]$$

Boundary conditions at the interface: y=b

$$\oint_C \vec{E} \cdot d\vec{l} = -\iint_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\widehat{U}_n \times \left(\vec{E}_i + \vec{E}_r\right) = \widehat{U}_n \times \vec{E}_t$$

$$\oiint_A \vec{D} \cdot d\vec{S} = \frac{Q_f}{\varepsilon_0} = 0$$

$$\widehat{U}_n \cdot \epsilon_i \left(\vec{E}_i + \vec{E}_r\right) = \widehat{U}_n \cdot \epsilon_t \vec{E}_t$$

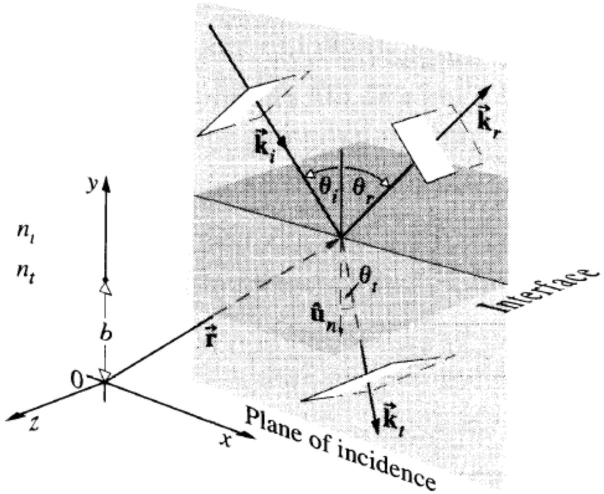


Figure 4.38 Plane waves incident on the boundary between two homogeneous, isotropic, lossless dielectric media.

Reflection and Refraction Laws

$$\vec{E}_i = \vec{E}_{i0} \exp[i(\vec{k}_i \cdot \vec{r} - \omega t)]$$

$$\vec{E}_r = \vec{E}_{r0} \exp[i(\vec{k}_r \cdot \vec{r} - \omega t + \varphi_r)]$$

$$\vec{E}_t = \vec{E}_{t0} \exp[i(\vec{k}_t \cdot \vec{r} - \omega t + \varphi_t)]$$

$$\hat{U}_n \times \left(\vec{E}_i + \vec{E}_r\right)|_{y=b} = \hat{U}_n \times \vec{E}_t|_{y=b}$$

$$\hat{U}_n \times \left(\vec{E}_{i0} \exp[i(\vec{k}_i \cdot \vec{r} - \omega t)] + \vec{E}_{r0} \exp[i(\vec{k}_r \cdot \vec{r} - \omega t + \varphi_r)]\right)|_{y=b}$$

$$= \hat{U}_n \times \vec{E}_{t0} \exp[i(\vec{k}_t \cdot \vec{r} - \omega t + \varphi_t)]|_{y=b}$$

$$\left(\vec{k}_i \cdot \vec{r} - \omega t\right)|_{y=b} = \left(\vec{k}_r \cdot \vec{r} - \omega t + \varphi_r\right)|_{y=b} = \left(\vec{k}_t \cdot \vec{r} - \omega t + \varphi_t\right)|_{y=b}$$

$$\left(\vec{k}_i \cdot \vec{r}\right)|_{y=b} = \left(\vec{k}_r \cdot \vec{r} + \varphi_r\right)|_{y=b} = \left(\vec{k}_t \cdot \vec{r} + \varphi_t\right)|_{y=b}$$

$$n_i k_0 x \sin\theta_i + n_i k_0 b \cos\theta_i = n_i k_0 x \sin\theta_r - n_i k_0 b \cos\theta_r + \varphi_r = n_t k_0 x \sin\theta_t + n_t k_0 b \cos\theta_t + \varphi_t$$

Reflection and Refraction Laws

$n_i k_0 x \sin\theta_i = n_i k_0 x \sin\theta_r = n_t k_0 x \sin\theta_t$

$\theta_i = \theta_r$

$n_i \sin\theta_i = n_t \sin\theta_t$

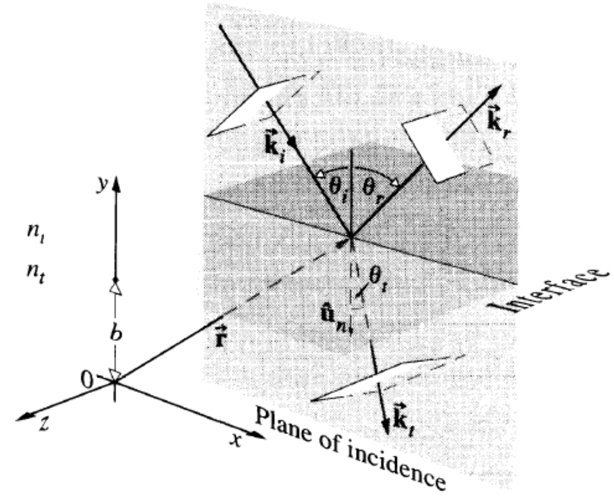


Figure 4.38 Plane waves incident on the boundary between two homogeneous, isotropic, lossless dielectric media.

$n_i k_0 b \cos\theta_i = -n_i k_0 b \cos\theta_r + \varphi_r = n_t k_0 b \cos\theta_t + \varphi_t$

Fresnel Equations: Amplitudes

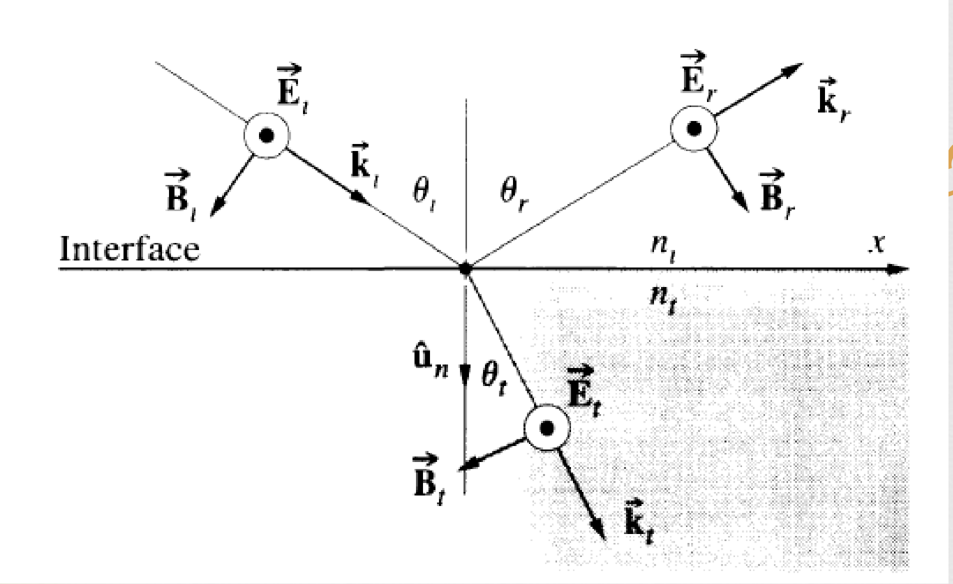
$$\hat{U}_n \times \left(\vec{E}_i + \vec{E}_r\right)|_{y=b} = \hat{U}_n \times \vec{E}_t|_{y=b}$$

$$\hat{U}_n \cdot \epsilon_i \left(\vec{E}_i + \vec{E}_r\right)|_{y=b} = \hat{U}_n \cdot \epsilon_t \vec{E}_t|_{y=b}$$

$$\hat{U}_n \times \left(\vec{E}_{i0} + \vec{E}_{r0}\right) = \hat{U}_n \times \vec{E}_{t0}$$

$$\hat{U}_n \cdot \epsilon_i \left(\vec{E}_{i0} + \vec{E}_{r0}\right) = \hat{U}_n \cdot \epsilon_t \vec{E}_{t0}$$

Case 1: E perpendicular to the plane-of-incidence



$$\hat{U}_n \times \left(\vec{E}_{i0} + \vec{E}_{r0}\right) = \hat{U}_n \times \vec{E}_{t0}$$

$$\boxed{E_{i0} + E_{r0} = E_{t0}}$$

$$-\frac{1}{\mu_i} B_{i0} \cos\theta_i + \frac{1}{\mu_i} B_{r0} \cos\theta_r = -\frac{1}{\mu_t} B_{r0} \cos\theta_t$$

$$-\frac{E_{i0}}{\mu_i v_i} \cos\theta_i + \frac{E_{r0}}{\mu_i v_i} \cos\theta_i = -\frac{E_{t0}}{\mu_t v_t} \cos\theta_t$$

$$\frac{1}{\mu_i v_i}(E_{i0} - E_{r0}) \cos\theta_i = \frac{1}{\mu_t v_t} E_{t0} \cos\theta_t$$

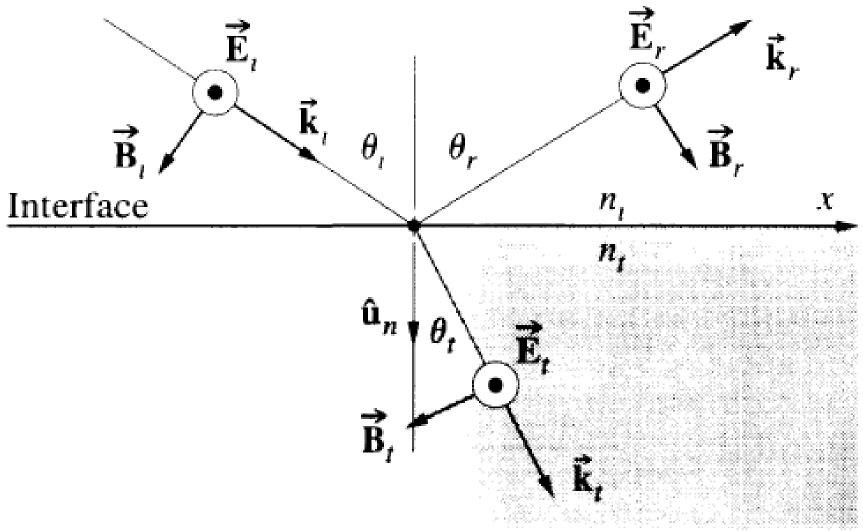
$$\boxed{\frac{n_i}{\mu_i}(E_{i0} - E_{r0}) \cos\theta_i = \frac{n_t}{\mu_t} E_{t0} \cos\theta_t}$$

$$\oint_C \vec{H} \cdot d\vec{l} = \iint_A \left(\vec{J}_f + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$$

$$\hat{U}_n \times \left(\vec{H}_{i0} + \vec{H}_{r0}\right) = \hat{U}_n \times \vec{H}_{t0}$$

$$\hat{U}_n \times \frac{1}{\mu_i}\left(\vec{B}_{i0} + \vec{B}_{r0}\right) = \frac{1}{\mu_t} \hat{U}_n \times \vec{B}_{t0}$$

Case 1: E perpendicular to the plane-of-incidence



$$\boxed{E_{i0} + E_{r0} = E_{t0}}$$

$$\boxed{\frac{n_i}{\mu_i}(E_{i0} - E_{r0}) \cos\theta_i = \frac{n_t}{\mu_t} E_{t0} \cos\theta_t}$$

$$\left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{\frac{n_i}{\mu_i}\cos\theta_i - \frac{n_t}{\mu_t}\cos\theta_t}{\frac{n_i}{\mu_i}\cos\theta_i + \frac{n_t}{\mu_t}\cos\theta_t} \tag{4.32}$$

and

$$\left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2\frac{n_i}{\mu_i}\cos\theta_i}{\frac{n_i}{\mu_i}\cos\theta_i + \frac{n_t}{\mu_t}\cos\theta_t} \tag{4.33}$$

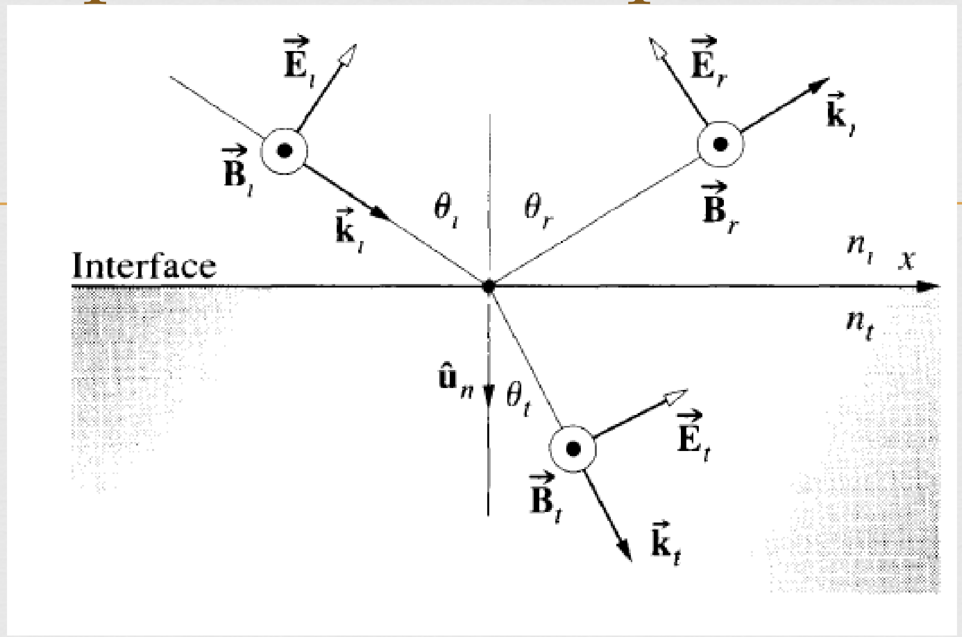
$$\mu_i \approx \mu_t \approx \mu_0$$

$$r_\perp \equiv \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \tag{4.34}$$

and

$$t_\perp \equiv \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \tag{4.35}$$

Case 2: E parallel to the plane-of-incidence



$$r_{\|} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} \tag{4.40}$$

and

$$t_{\|} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} \tag{4.41}$$

Snell's Laws and Fresnel Eqs.

$\theta_i = \theta_r$

$n_i \sin\theta_i = n_t \sin\theta_t$

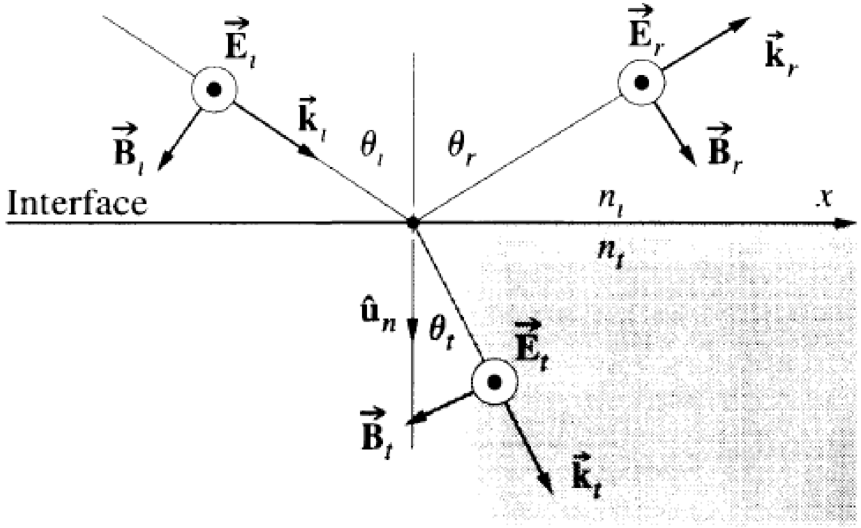
$$r_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \tag{4.42}$$

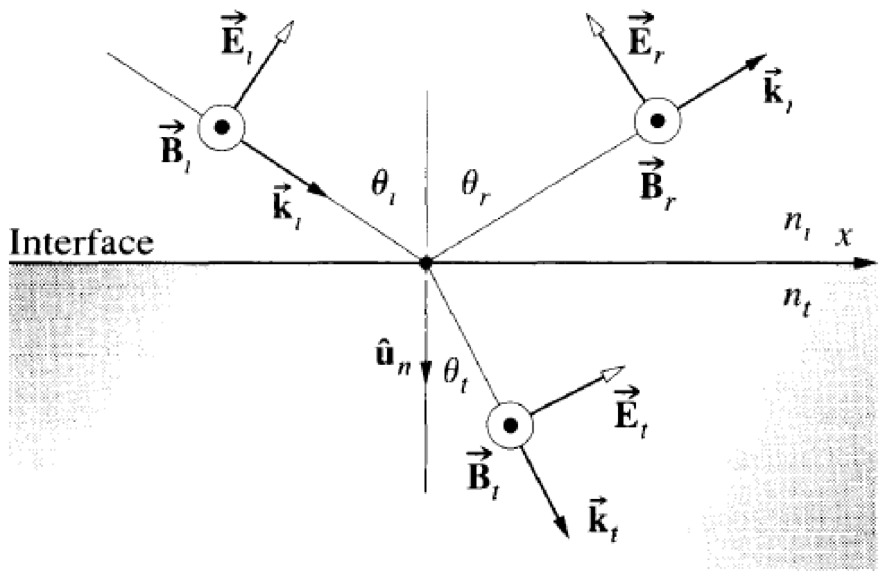
$$r_\parallel = +\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \tag{4.43}$$

$$t_\perp = +\frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i + \theta_t)} \tag{4.44}$$

$$t_\parallel = +\frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)} \tag{4.45}$$

Amplitude Coefficients





$$r_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \tag{4.42}$$

$$r_\parallel = +\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \tag{4.43}$$

$$t_\perp = +\frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i + \theta_t)} \tag{4.44}$$

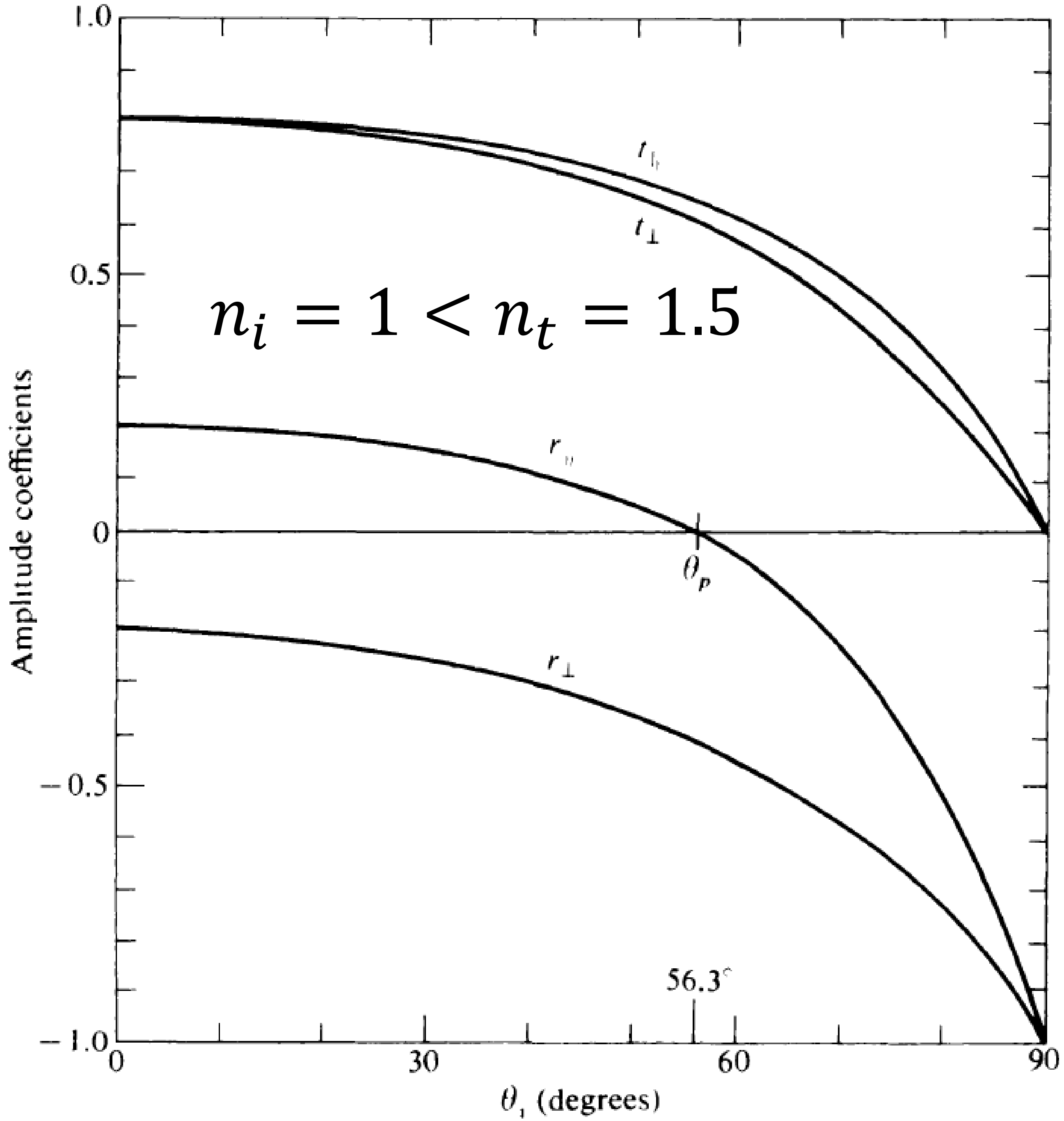
$$t_\parallel = +\frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)} \tag{4.45}$$

$n_i < n_t$ $\qquad$ $\theta_i > \theta_t$

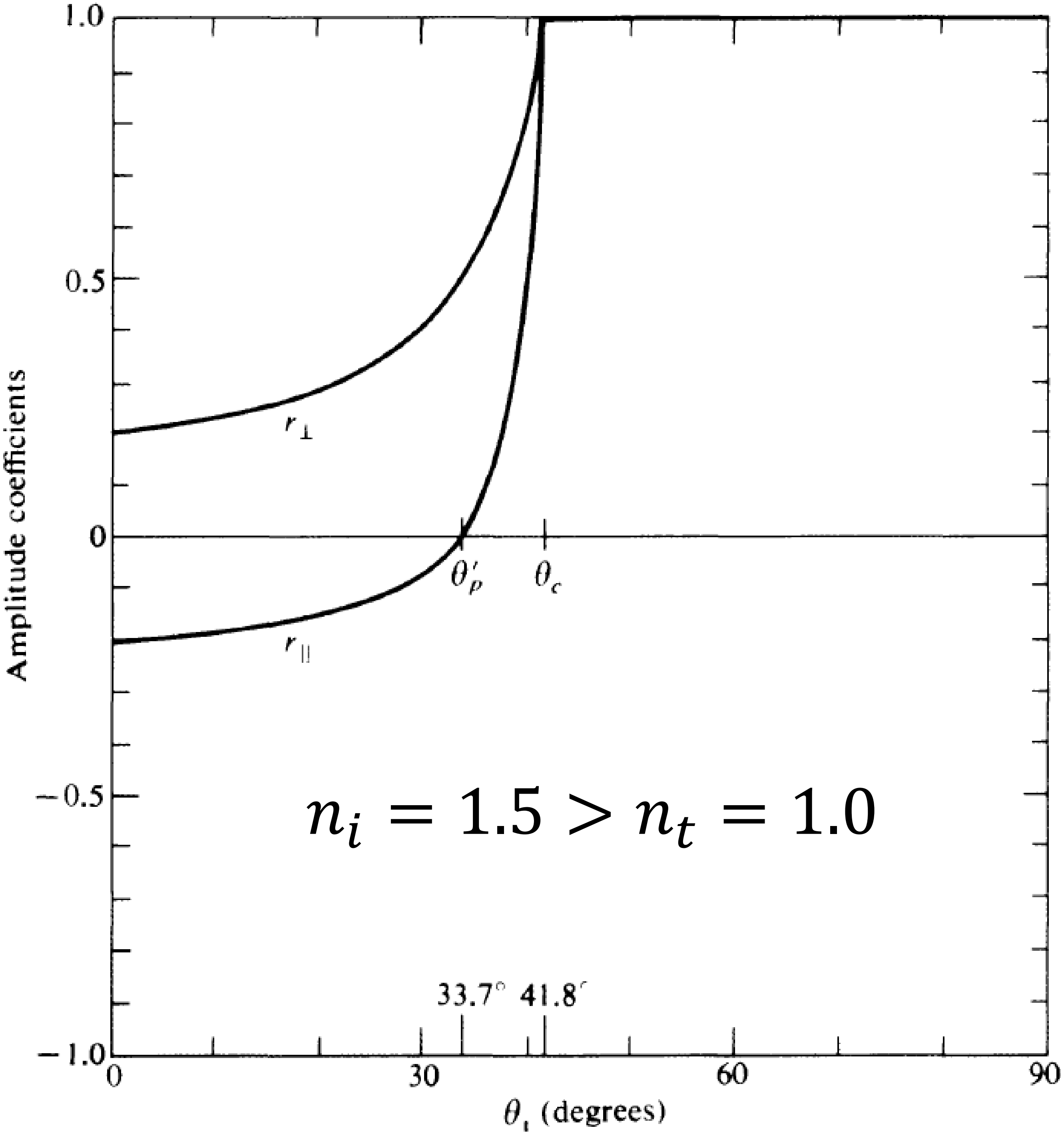
$r_\perp < 0$ $\qquad$ $r_{||} > 0?$

$\theta_i = 0$

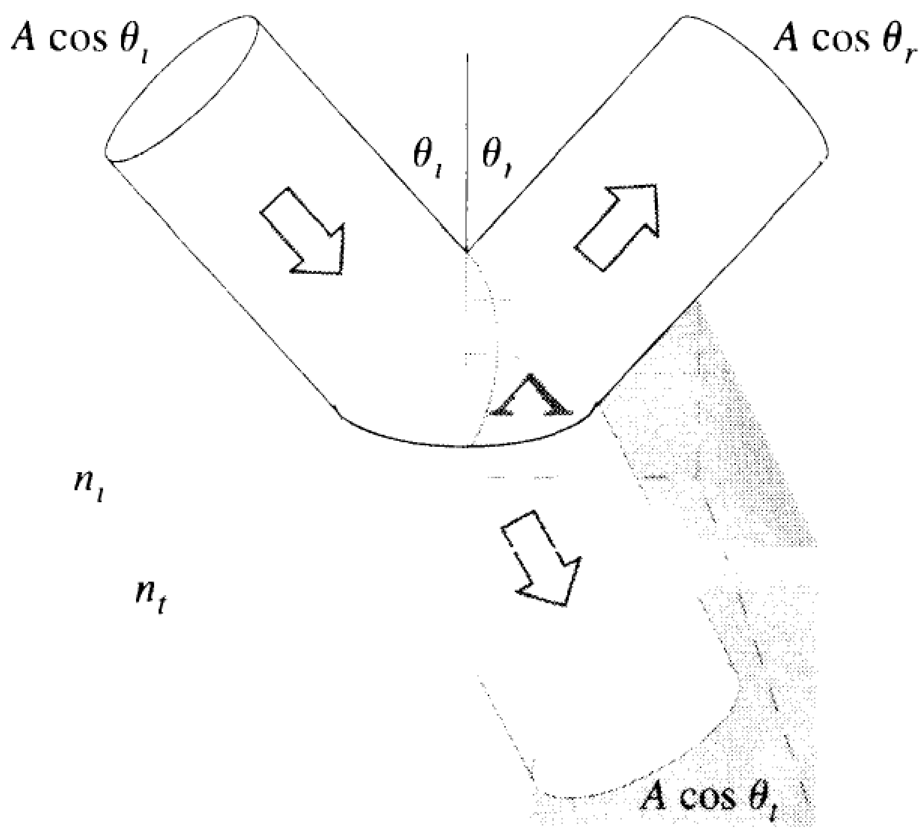
$$r_\perp = -r_{||} = \frac{n_i - n_t}{n_i + n_t}$$



$n_i = 1 < n_t = 1.5$
$t_\parallel$
$t_\perp$
$r_\parallel$
$r_\perp$
θ_p
56.3°
Amplitude coefficients
θ_1 (degrees)
1.0
0.5
0
−0.5
−1.0
0
30
60
90


1.0
0.5
0
−0.5
−1.0
Amplitude coefficients
$r_\perp$
$r_\parallel$
θ_p'
θ_c
$n_i = 1.5 > n_t = 1.0$
33.7° 41.8°
0
30
60
90
θ_i (degrees)

Reflectance & Transmittance



$$R \equiv \frac{I_r A \cos\theta_r}{I_i A \cos\theta_i} = \frac{I_r}{I_i}$$

$$R = \left(\frac{E_{0r}}{E_{0i}}\right)^2 = r^2$$

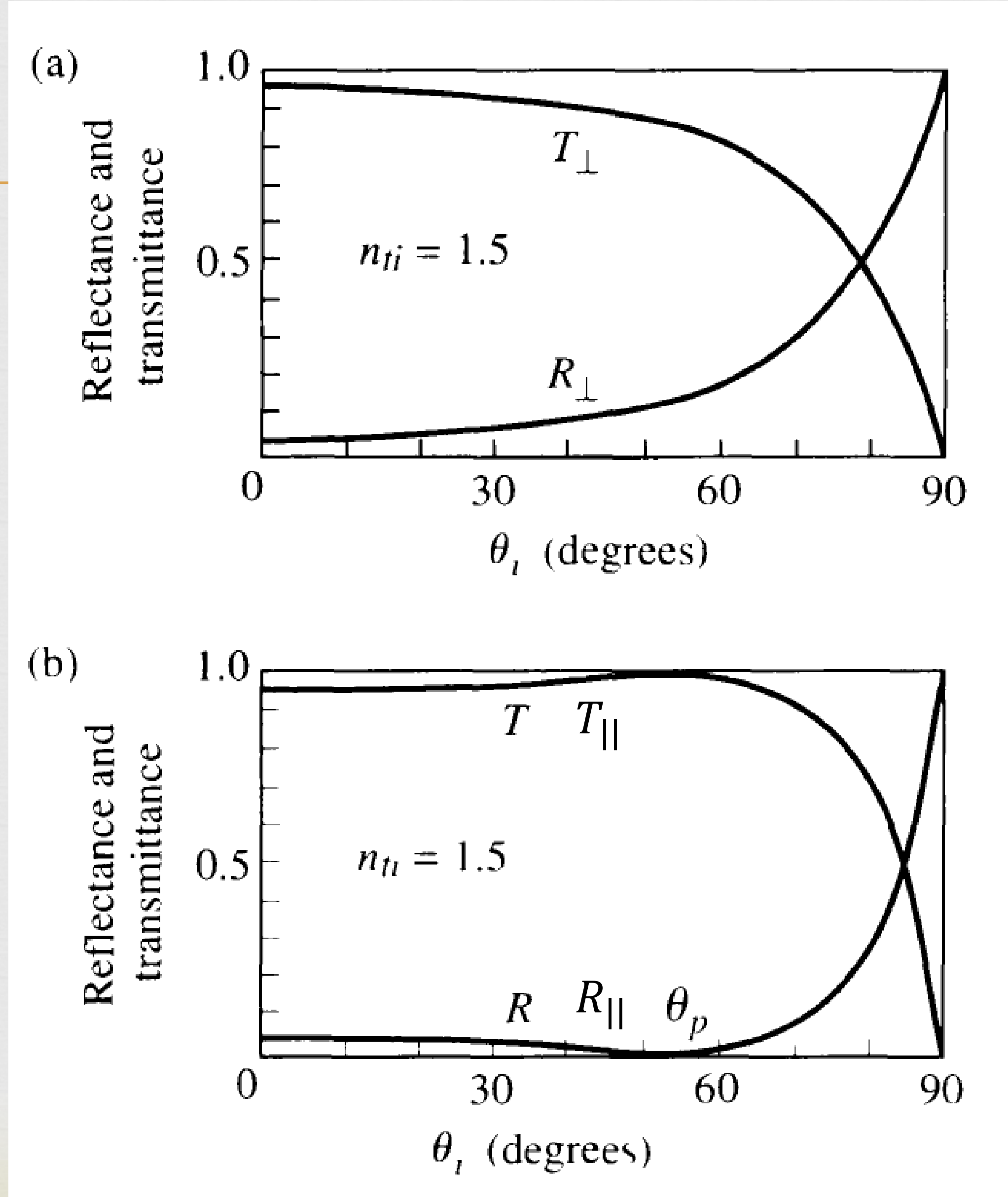
$$T \equiv \frac{I_t \cos\theta_t}{I_i \cos\theta_i}$$

$$R + T = 1$$

$$T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\left(\frac{E_{0t}}{E_{0i}}\right)^2 = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right)t^2$$

Reflectance & Transmittance

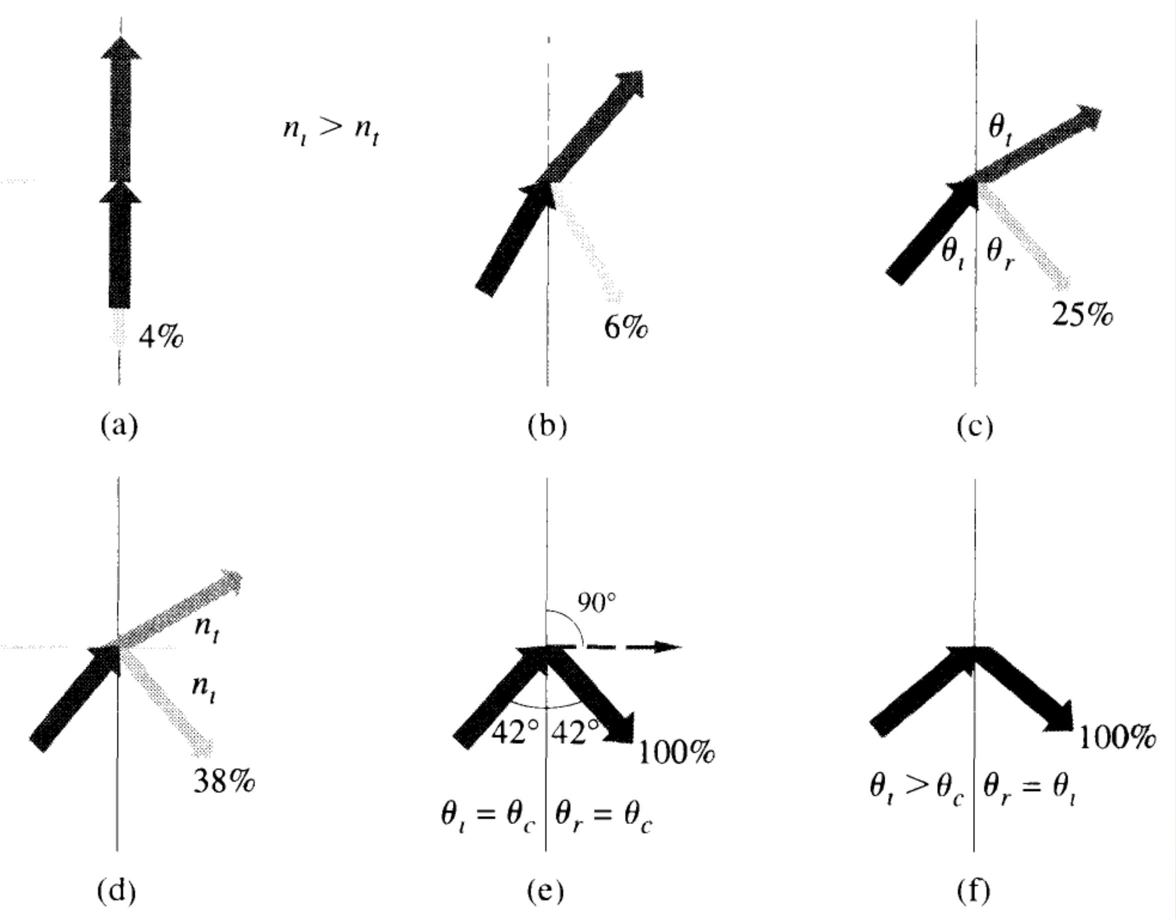
$$n_{ti} = \frac{n_t}{n_i}$$

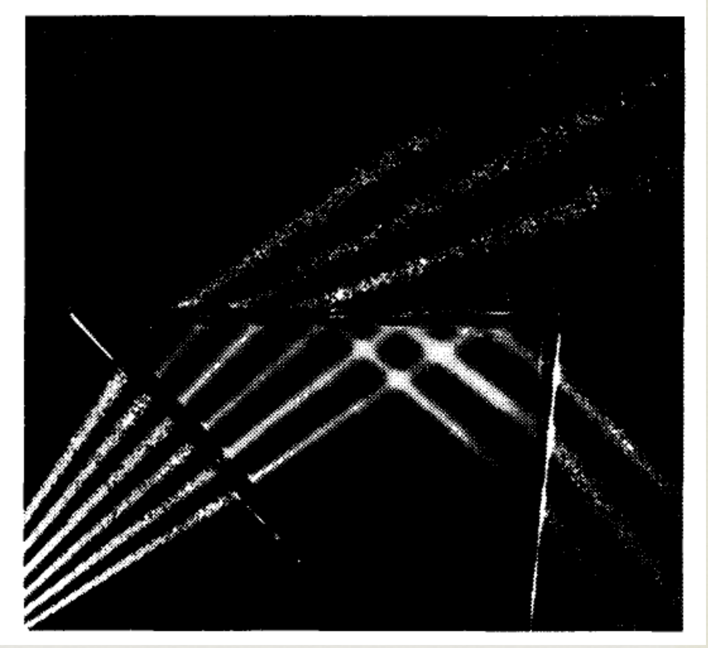


4.7 Total Internal Reflection

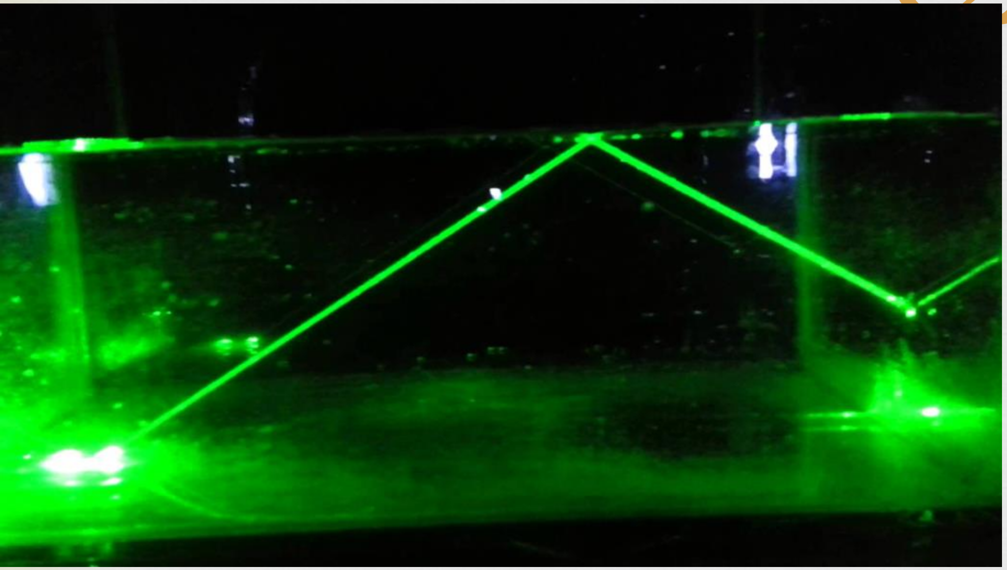
$n_i \sin\theta_i = n_t \sin\theta_t$

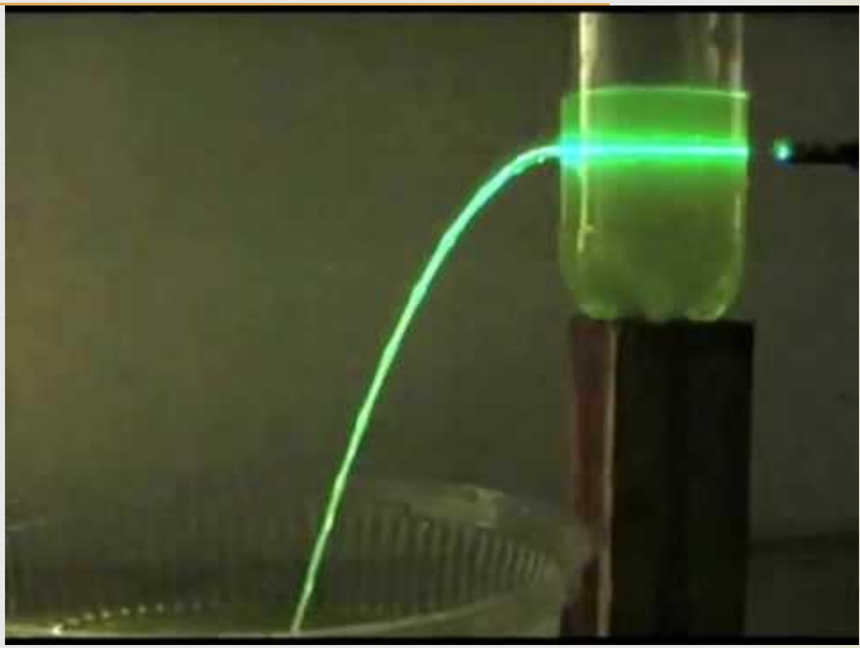
$$\sin\theta_t = \frac{n_i}{n_t}\sin\theta_i$$

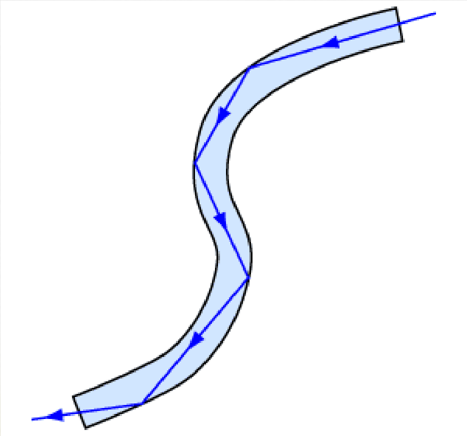




Total Internal reflection







4.7.1 Evanescent Wave