PHYS 3038 Optics
L4 EM Theory, Photons, & Light
Reading Material: Ch3.3-3.7

Shengwang Du

2015, the Year of Light

3.3 Energy & Momentum

Vacuum

$$u_E = \frac{\varepsilon_0}{2} E^2 \qquad u_B = \frac{1}{2\mu_0} B^2$$

For plane wave: $E = CB$

$$u = u_E + u_B = \varepsilon_0 E^2 = B^2 / \mu_0$$ Energy density

Energy flow density $S = uc = c\varepsilon_0 E^2 = c\varepsilon_0 EcB = c^2 \varepsilon_0 EB = \frac{1}{\mu_0} EB$

Poynting vector $\vec{S} = c^2 \varepsilon_0 \vec{E} \times \vec{B} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

Medium

$$u = \frac{1}{2}(\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B}) \qquad \vec{S} = \vec{E} \times \vec{H}$$

Average Energy and Power of EM Wave

$$\vec{E} = \vec{E}_0 \cos(\vec{k}\cdot\vec{r} - \omega t) = \mathrm{Re}\{e^{i(\vec{k}\cdot\vec{r}-\omega t)}\}$$

Optical frequency ~10^{15} Hz

$$u = \varepsilon_0 E^2 = \varepsilon_0 E_0^2 \cos^2(\vec{k}\cdot\vec{r} - \omega t) = \frac{1}{2}\varepsilon_0 E_0^2 \left[1 + \cos 2(\vec{k}\cdot\vec{r} - \omega t)\right]$$

$$= \frac{1}{2}\varepsilon_0 E_0^2 + \frac{1}{2}\varepsilon_0 E_0^2 \cos 2(\vec{k}\cdot\vec{r} - \omega t)$$

Time-averaged $$\langle u\rangle_T = \frac{1}{2}\varepsilon_0 E_0^2 + \frac{1}{2}\varepsilon_0 E_0^2 \langle \cos 2(\vec{k}\cdot\vec{r} - \omega t)\rangle_T = \frac{1}{2}\varepsilon_0 E_0^2$$

Intensity $$I = \langle S\rangle_T = c\langle u\rangle_T = \frac{1}{2}c\varepsilon_0 E_0^2$$ average energy per unit area and per unit time (W/m^2)

Power $$P = IA = \frac{1}{2}c\varepsilon_0 E_0^2 A$$ average energy through the area A per unit time (W)

3.3.3 Photons

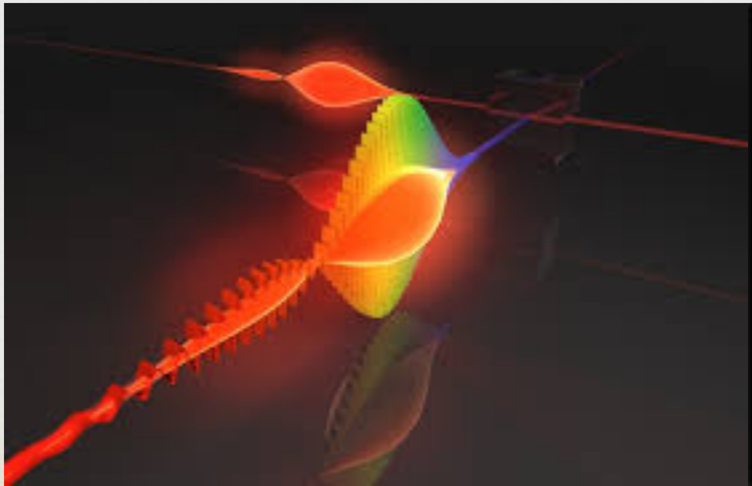
- Photon: particle (quantization) of EM field – chargeless and massless

Photon of a plane wave $(\vec{k}, \omega)$

Energy of a photon in the plane wave: $\hbar\omega = h\nu$

Momentum of a photon in the plane wave: $\hbar k = \dfrac{h}{\lambda}$

Mean photon flux $\Phi = \dfrac{P}{\hbar\omega}$



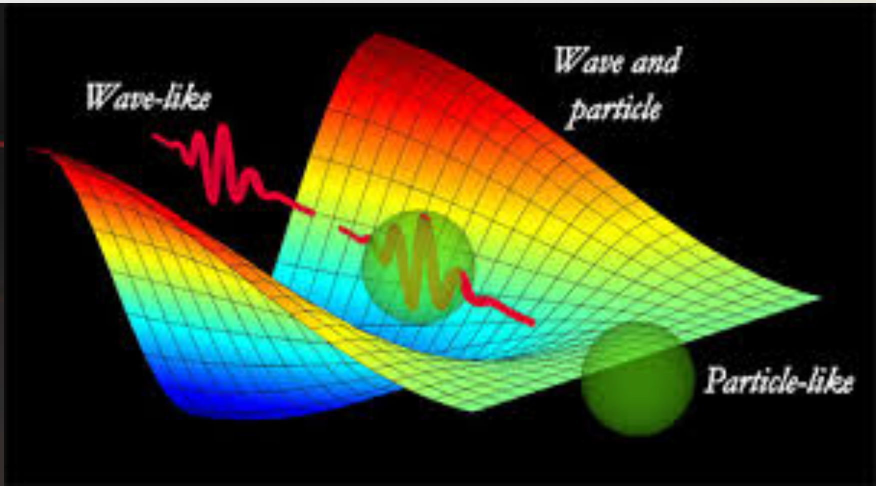
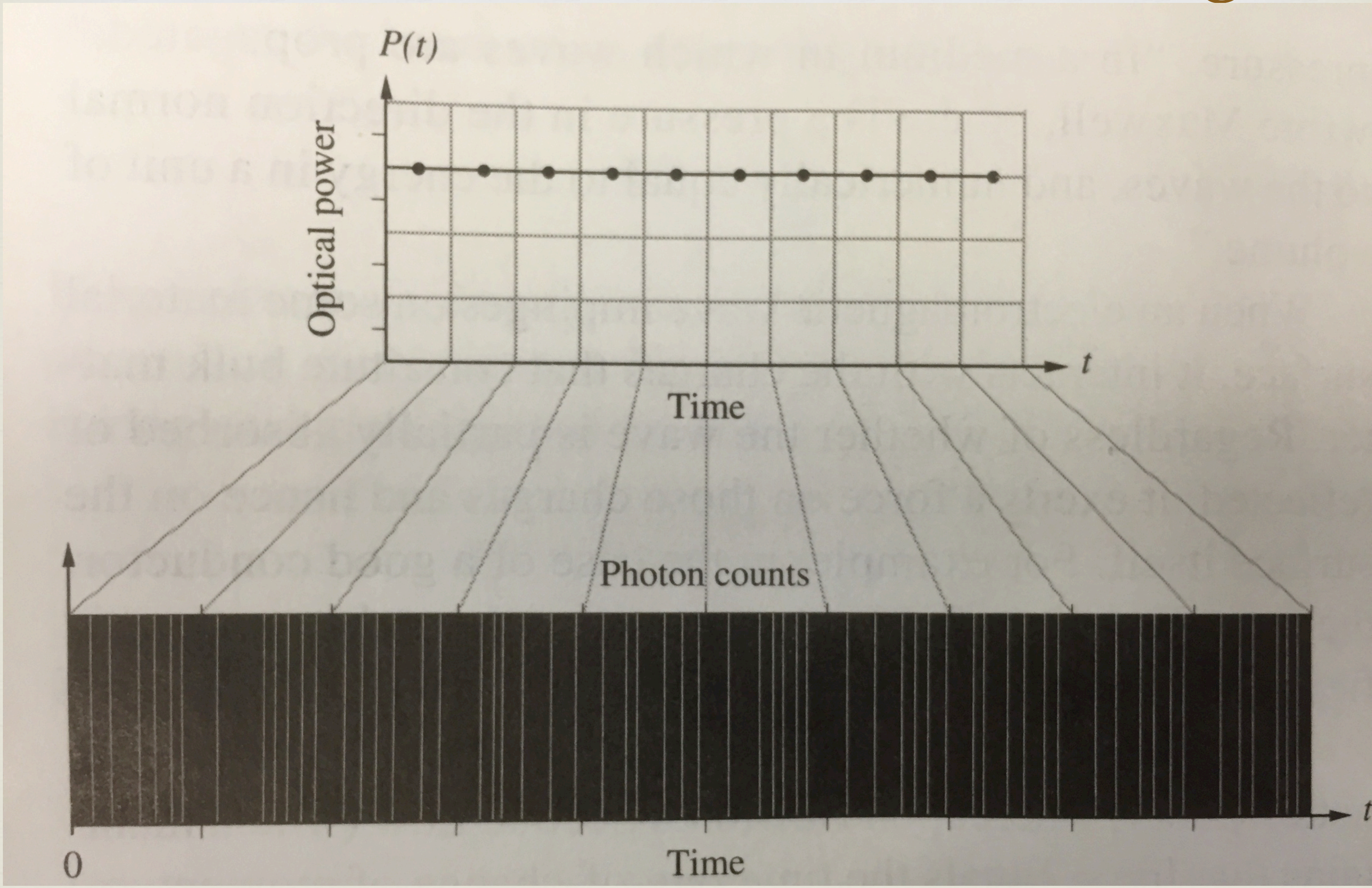


TABLE 3.1 The Mean Photon Flux Density for a Sampling of Common Sources

Light Source	Mean Photon Flux Density Φ/A in units of (photons/s·m^2)
Laserbeam (10 mW, He-Ne, focused to 20 μm)	10^{26}
Laserbeam (1 mW, He-Ne)	10^{21}
Bright sunlight	10^{18}
Indoor light level	10^{16}
Twilight	10^{14}
Moonlight	10^{12}
Starlight	10^{10}

Photon Counts of a coherent light



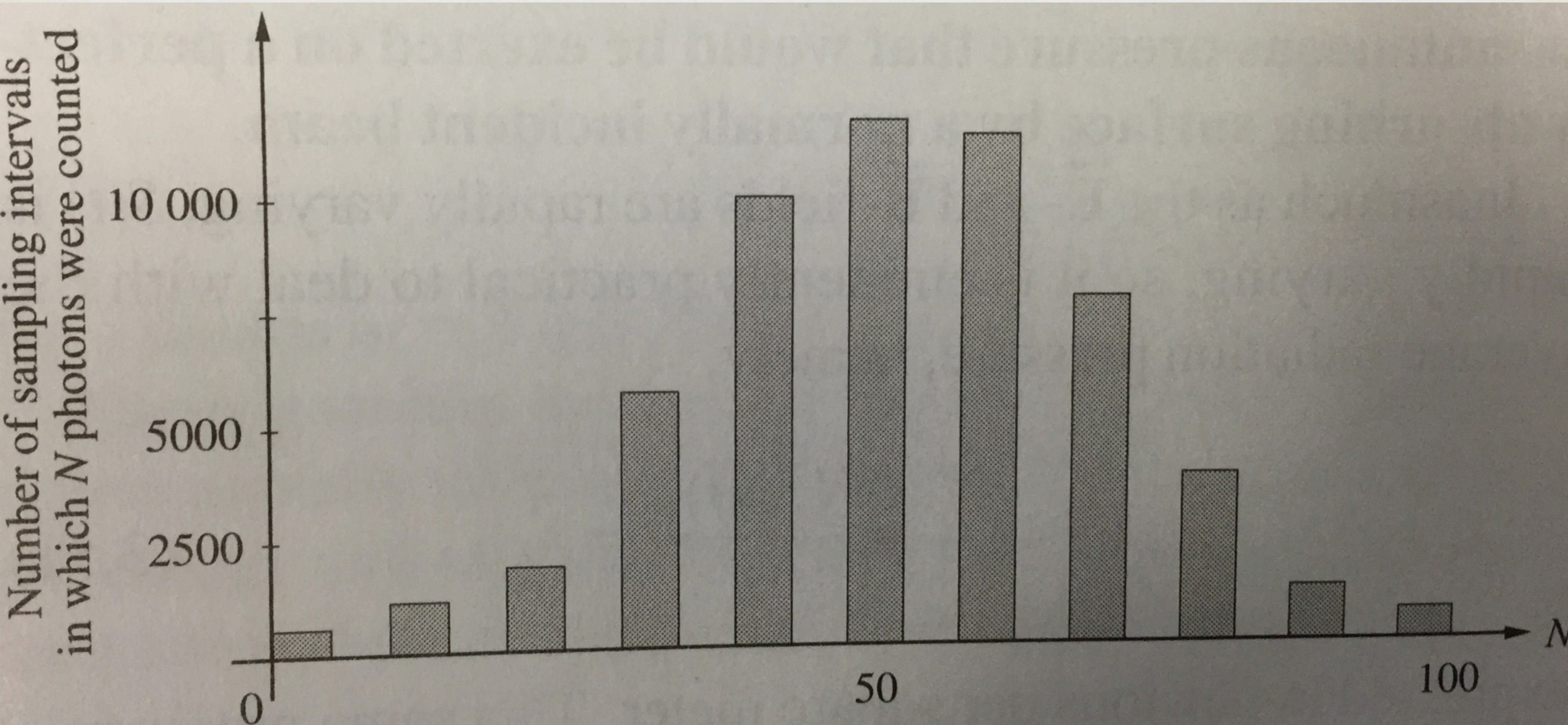


Figure 3.23 A typical histogram showing the probability or photon-count distribution for a beam of constant irradiance.

EM Wave VS Photon

$$\vec{E} = \vec{E}_0(\vec{r},t)\cos(\vec{k}\cdot\vec{r} - \omega t) = \mathrm{Re}\{\vec{E}_0(\vec{r},t)e^{i(\vec{k}\cdot\vec{r}-\omega t)}\}$$

$$\langle u \rangle_T = \frac{1}{2}\varepsilon_0 E_0^2(\vec{r},t) = N(\vec{r},t)\hbar\omega$$

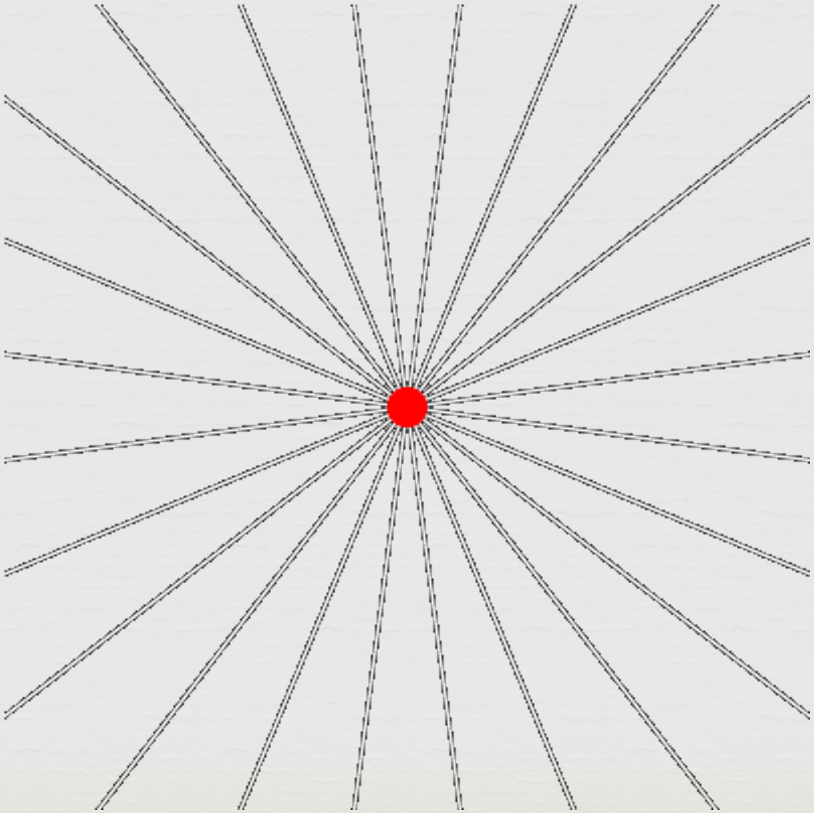
$$N(\vec{r},t) = \frac{\varepsilon_0 E_0^2(\vec{r},t)}{2\hbar\omega}$$

Photon # density ~ $E_0^2(\vec{r},t)$

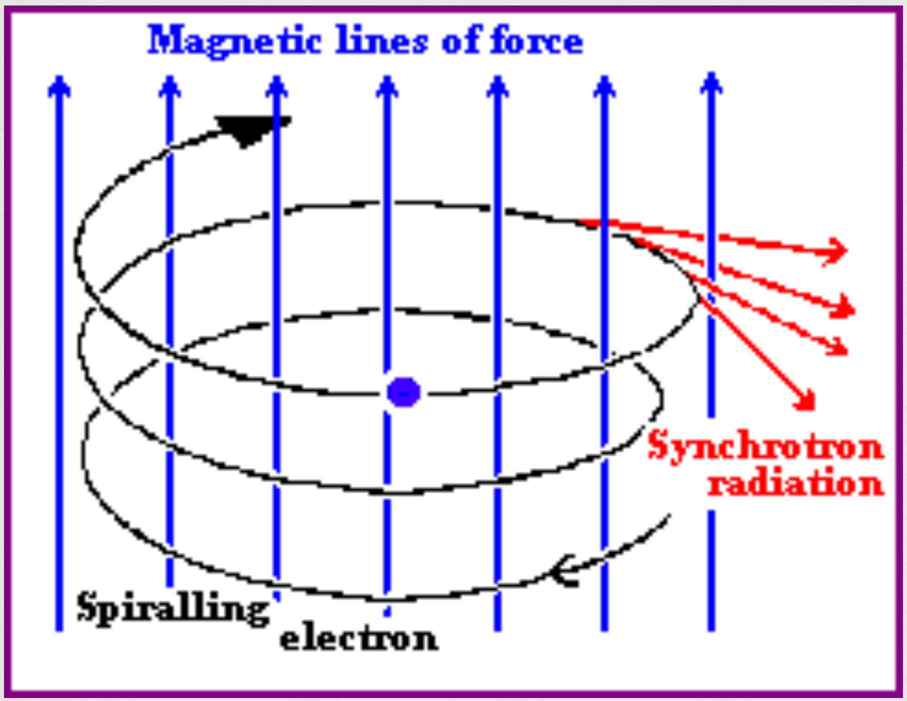
Read texbook

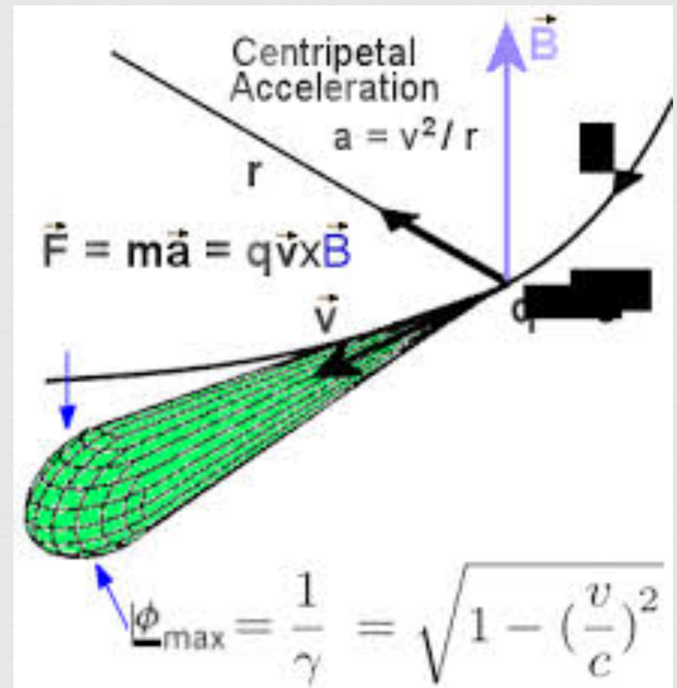
3.4 Radiation: Classical EM

- Accelerating charge → Radiation



Synchrotron Radiation

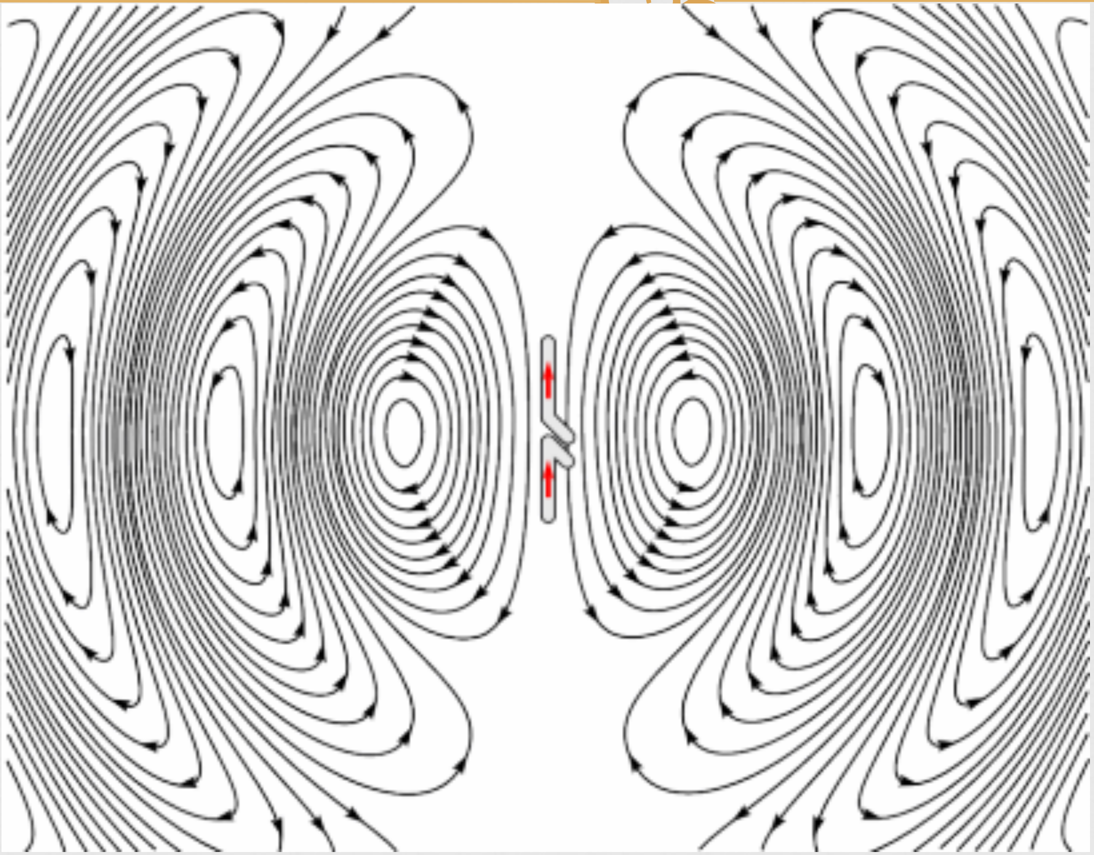




Synchrotron Light Source

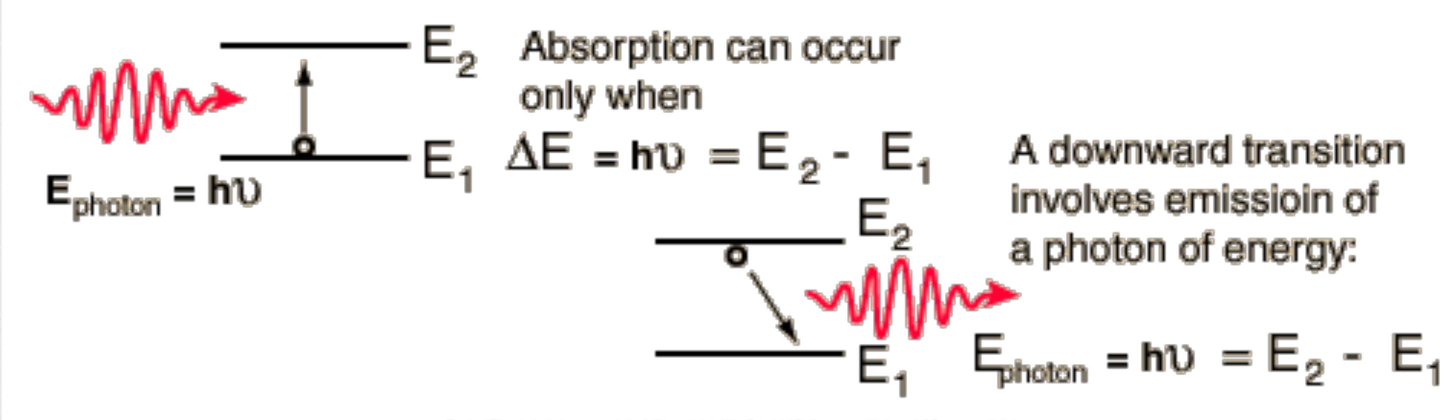
https://en.wikipedia.org/wiki/Synchrotron_light_source

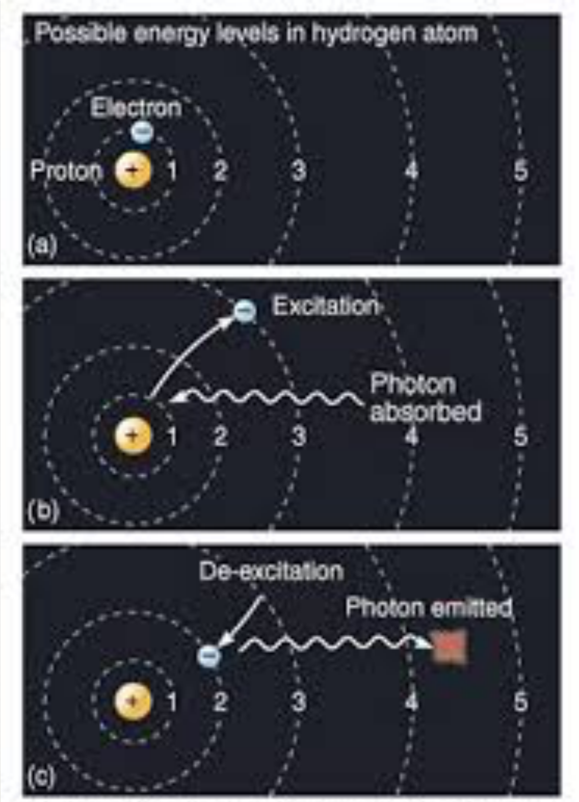
Electric Dipole Radiation



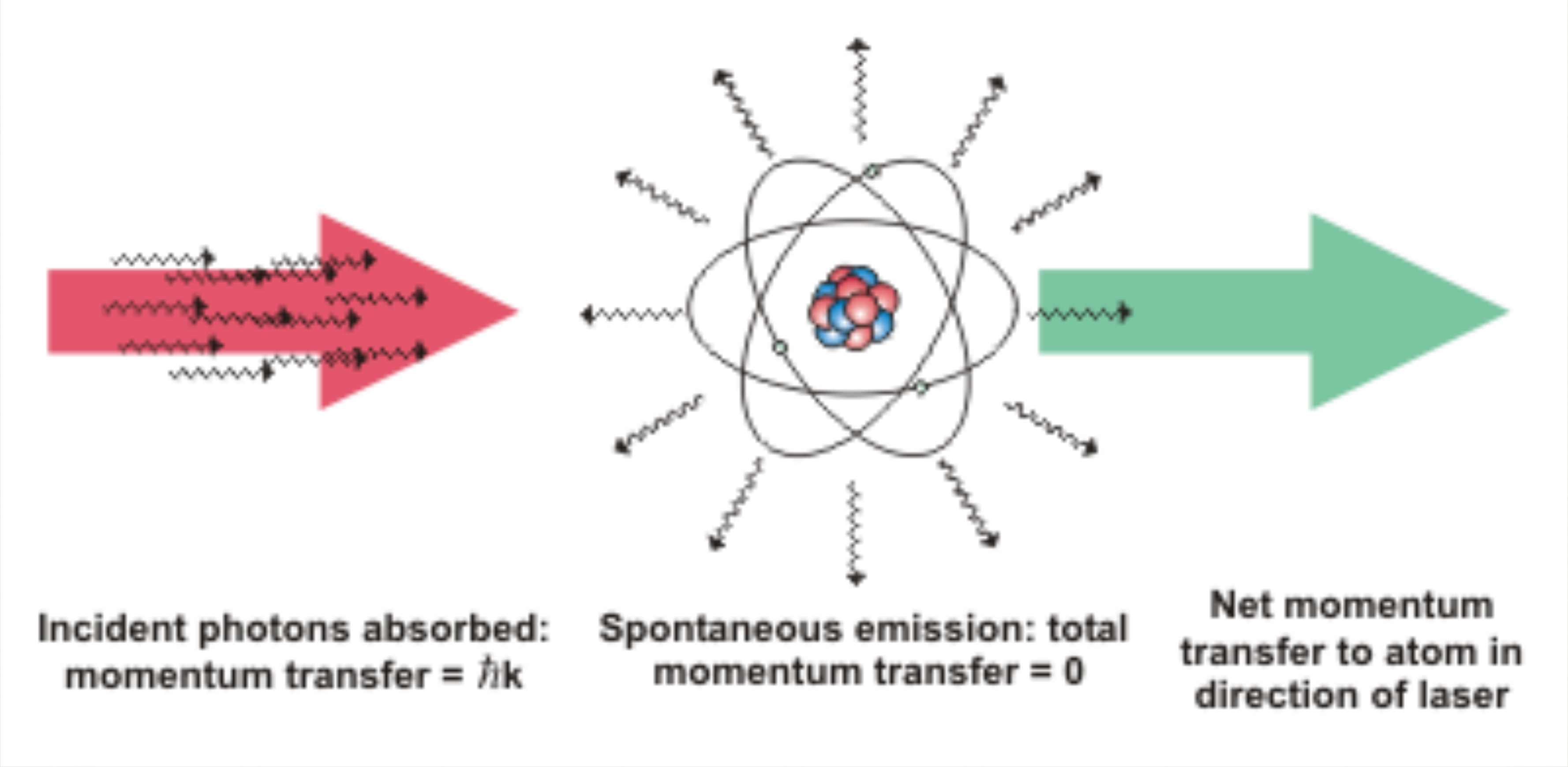
More on: https://en.wikipedia.org/wiki/Dipole_antenna

Quantum Physics of Radiation



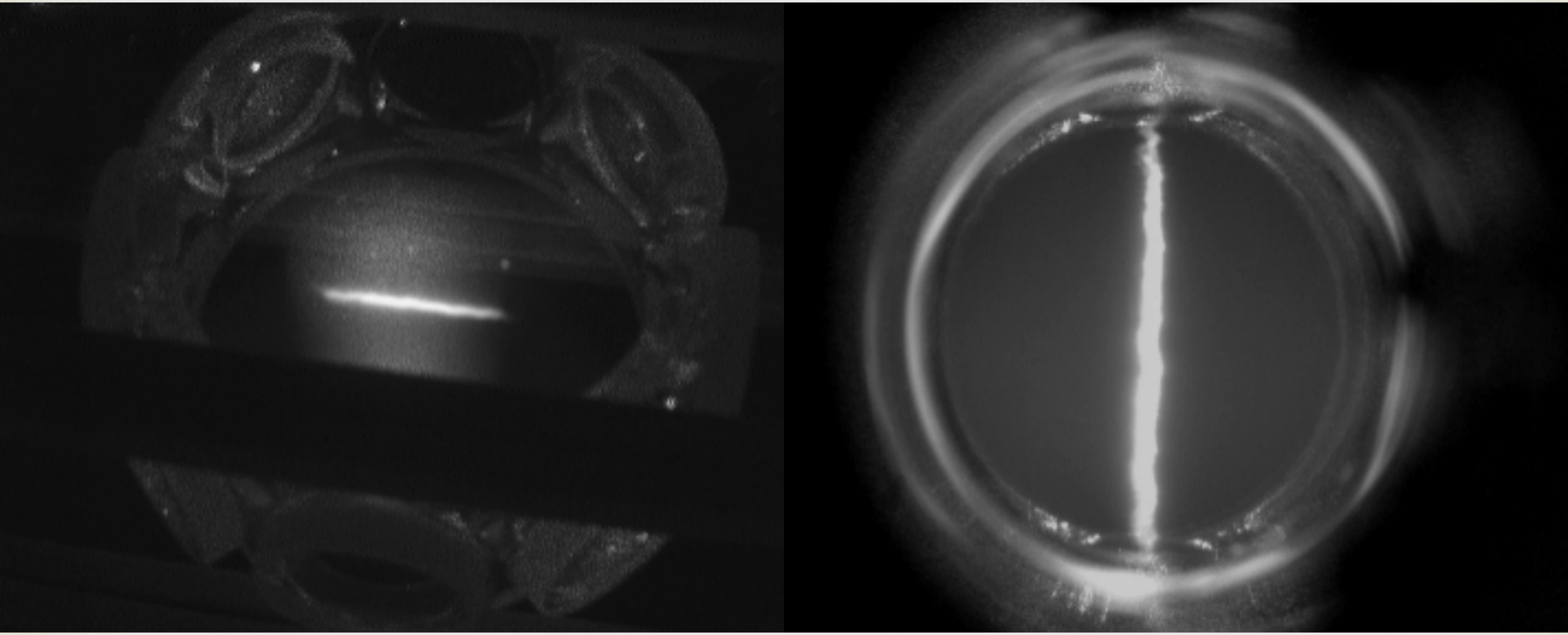


Laser Cooling and Trapping



1997 Nobel Prize in Physics

2D ^{85}Rb MOT @ HKUST



Number of atoms: ~10^8
Temperature: ~10 uK

Light in Bulk (Dielectric) Matter

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu}$$

$$\varepsilon_0 \Rightarrow \varepsilon = \varepsilon(\omega)$$

Dispersion

$$\mu_0 \Rightarrow \mu = \mu(\omega)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

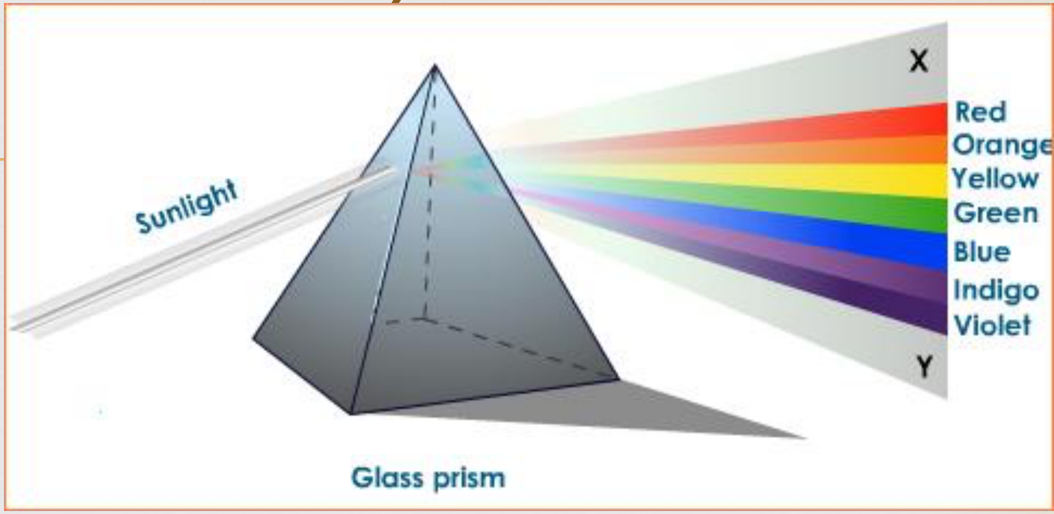
$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho_f$$

$$\nabla \cdot \vec{B} = 0$$

For most material (nonmagnetic) $\mu = \mu(\omega) \cong \mu_0$

Dispersion (Dielectric)



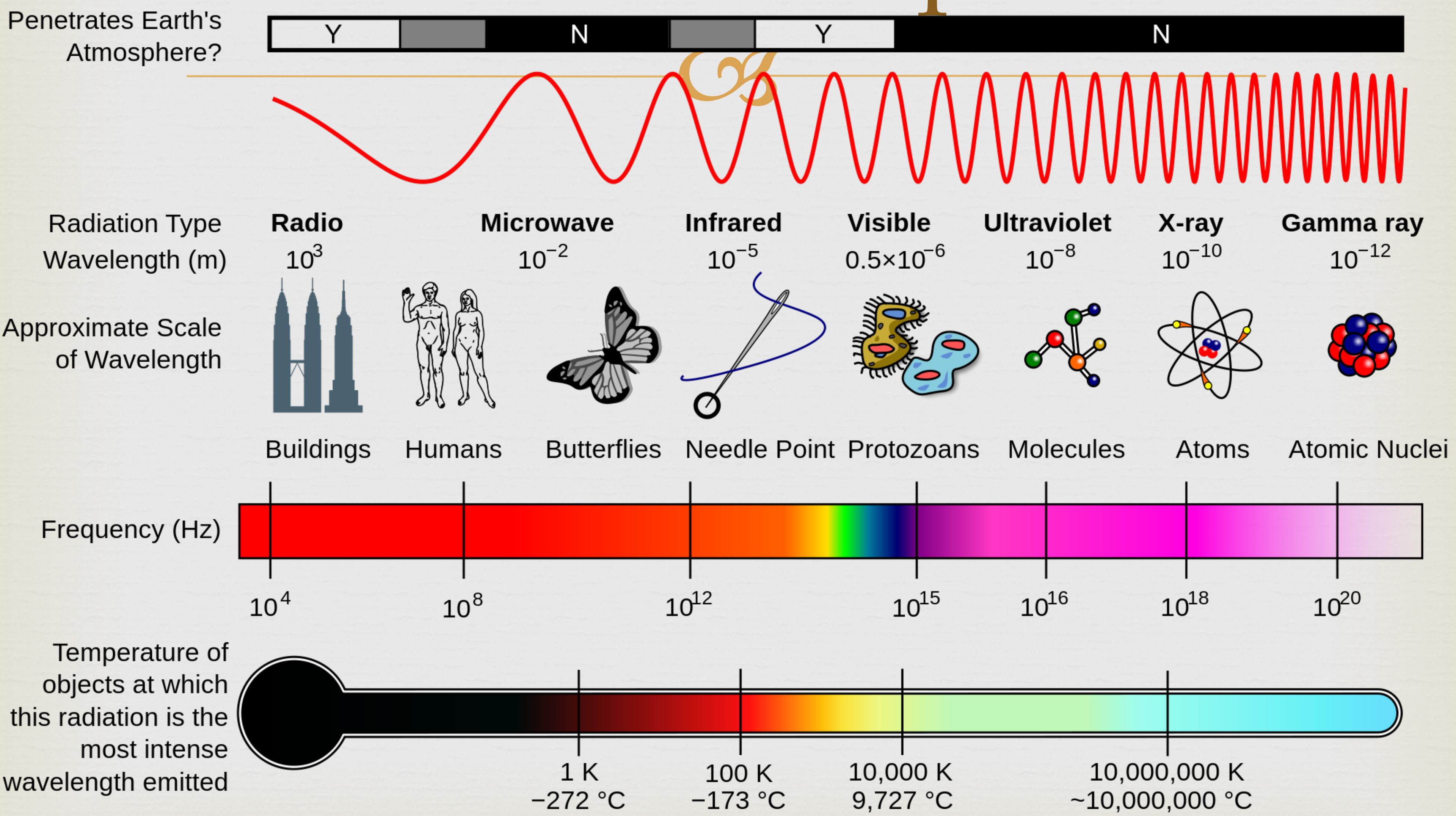
$$v(\omega) = \frac{1}{\sqrt{\varepsilon(\omega)\mu(\omega)}} = \frac{c}{n(\omega)}$$

$$n(\omega) = \sqrt{\frac{\varepsilon(\omega)\mu(\omega)}{\varepsilon_0\mu_0}} = \sqrt{K_E(\omega)K_M(\omega)}$$

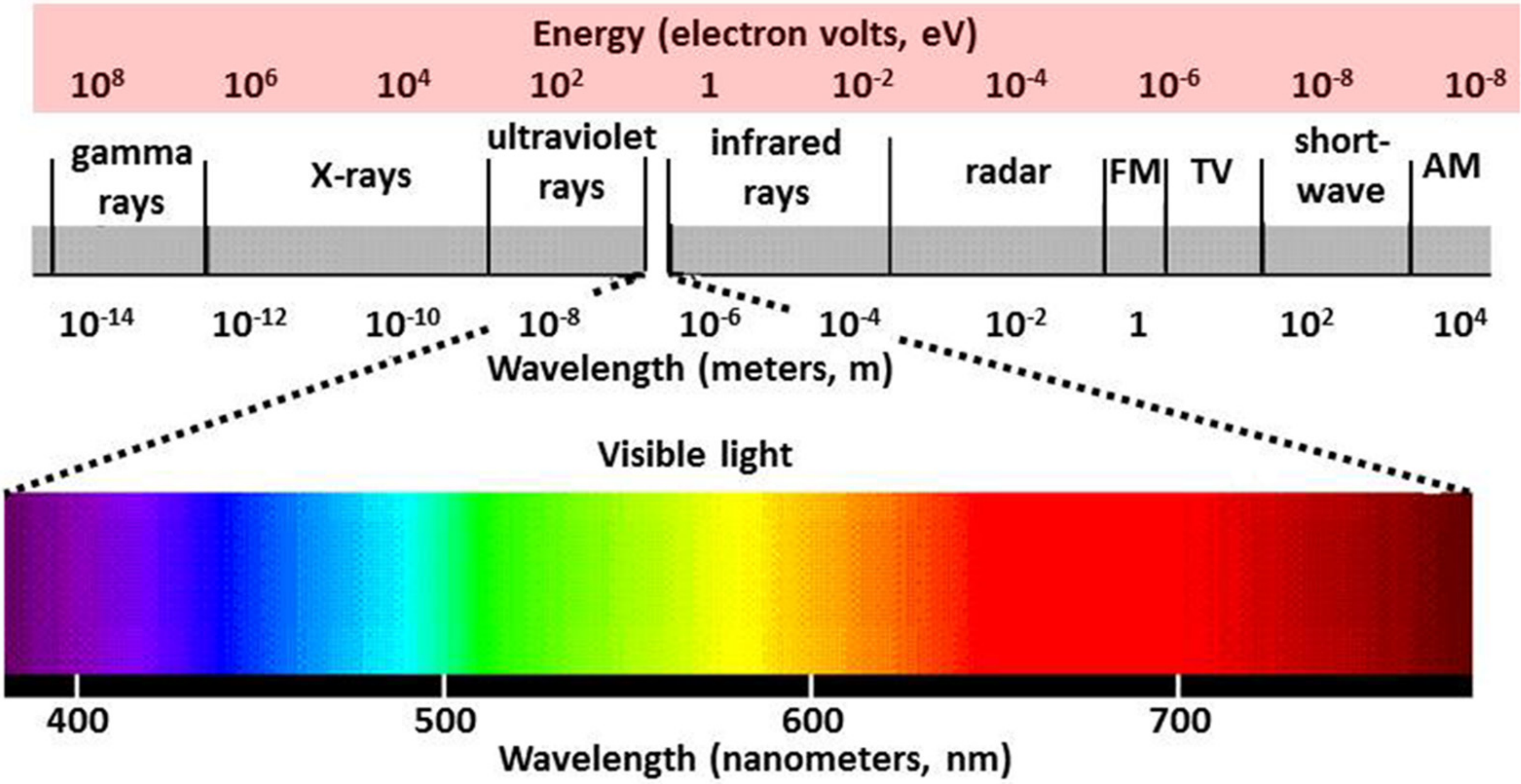
$$K_E(\omega) = \sqrt{\frac{\varepsilon(\omega)}{\varepsilon_0}}$$ Dielectric constant

$$K_M(\omega) = \sqrt{\frac{\mu(\omega)}{\mu_0}} \cong 1$$

3.6 EM-Photon Spectrum



EM Spectrum



Energy (electron volts, eV)
10^8
10^6
10^4
10^2
1
10^-2
10^-4
10^-6
10^-8
10^-8
gamma rays
X-rays
ultraviolet rays
infrared rays
radar
FM
TV
short-wave
AM
10^-14
10^-12
10^-10
10^-8
10^-6
10^-4
10^-2
1
10^2
10^4
Wavelength (meters, m)
Visible light
400
500
600
700
Wavelength (nanometers, nm)

3.7 Quantum Field Theory

- QFT: wave-particle duality
  - Wave is the field of its particle
  - Particle of the particle of its field.

- QED (Quantum Electrodynamics): Quantum theory of EM field and light-matter interaction.