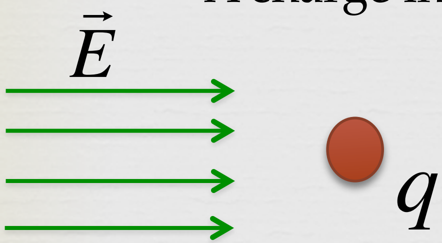
PHYS 3038 Optics
L3 EM Theory, Photons, & Light
Reading: Ch3.1-3.2

Shengwang Du

2015, the Year of Light

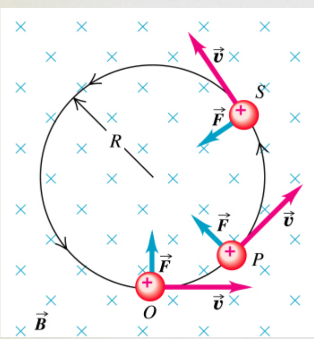
3.1 Basic Laws of EM Theory

A charge in the EM field



$$\vec{F}_E = q\vec{E}$$

What is the EM field? It can be sensed by an electric charge.

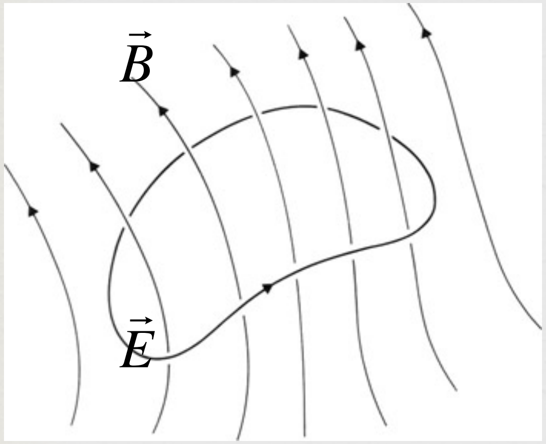


$$\vec{F}_M = q\vec{v} \times \vec{B}$$

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

3.1.1 Faraday Induction Law

"Convert magnetism into electricity", Michael Faraday, 1822



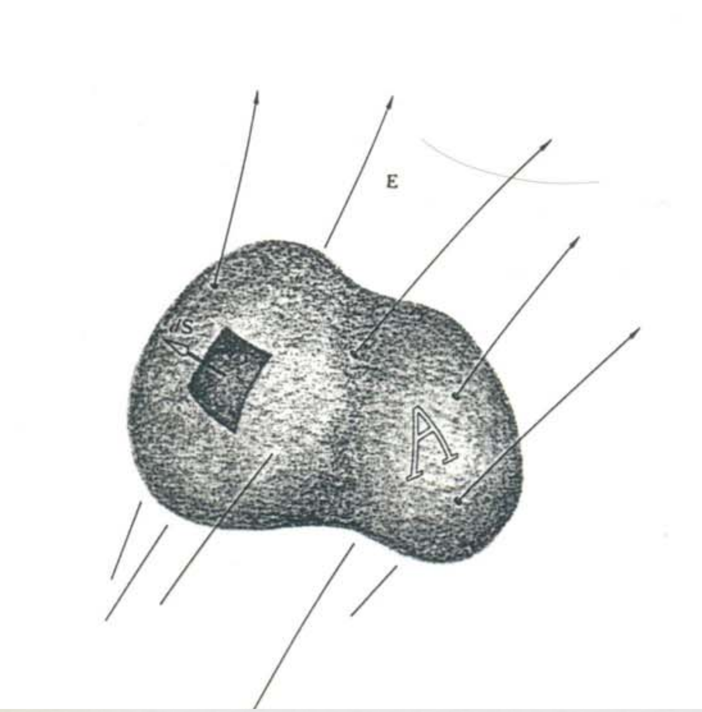
$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\iint_A \vec{B}\cdot d\vec{S} = -\iint_A \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

Stokes' theorem

$$\boxed{\oint_C \vec{E}\cdot d\vec{l} = \iint_A (\nabla\times\vec{E})\cdot d\vec{S}}$$

$$\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

3.1.3 Gauss's Law - Electric



$$\oiint_A \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\iiint_V \rho\, dV$$

Stokes theorm

$$\boxed{\oiint_A \vec{E}\cdot d\vec{S} = \iiint_V (\nabla\cdot\vec{E})\, dV}$$

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

3.1.3 Gauss's Law - Magnetic

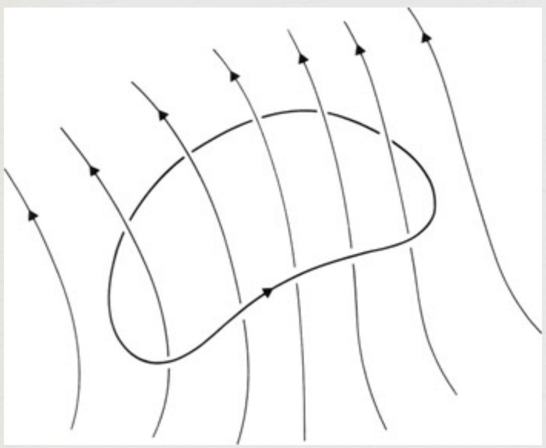
There is no magnetic charge (monopole)!

$$\oiint_A \vec{B} \cdot d\vec{S} = 0$$

$$\nabla \cdot \vec{B} = 0$$

3.1.4 Ampere's Law

"Convert electricity into magnetism", James Maxwell



$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 \iint_A \left(\vec{J} + \varepsilon_0 \underbrace{\frac{\partial \vec{E}}{\partial t}}_{} \right)\cdot d\vec{S}$$

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

Maxwell Equations - General

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

Question: Do the above equations apply for vacuum or/and medium?

Maxwell Equations in a Medium

Polarization $\vec{P}$

Magnetization $\vec{M}$

Bounded charge density $\rho_b = -\nabla \cdot \vec{P}$

Bounded current density $\vec{J}_b = \nabla \times \vec{M} + \dfrac{\partial \vec{P}}{\partial t}$

$$\rho = \rho_f + \rho_b$$

$$\vec{J} = \vec{J}_f + \vec{J}_b$$

$$\boxed{\begin{aligned} &\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ &\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \\ &\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \\ &\nabla \cdot \vec{B} = 0 \end{aligned}}$$

Maxwell Equations in a Medium

Polarization $\vec{P}$

Magnetization $\vec{M}$

Bounded charge density $\rho_b = -\nabla \cdot \vec{P}$

Bounded current density $\vec{J}_b = \nabla \times \vec{M} + \dfrac{\partial \vec{P}}{\partial t}$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho_f$$

$$\nabla \cdot \vec{B} = 0$$

Maxwell Equations in a Linear Medium

Polarization $\vec{P}$

Magnetization $\vec{M}$

Bounded charge density $\rho_b = -\nabla \cdot \vec{P}$

Bounded current density $\vec{J}_b = \nabla \times \vec{M} + \dfrac{\partial \vec{P}}{\partial t}$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu \left(\vec{J}_f + \varepsilon \frac{\partial \vec{E}}{\partial t} \right)$$

$$\nabla \cdot \vec{E} = \frac{\rho_f}{\varepsilon}$$

$$\nabla \cdot \vec{B} = 0$$

Maxwell Equations
(Vacuum, No Source)

$\rho = 0 \qquad \vec{J} = 0$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times (\nabla \times) = \nabla (\nabla \cdot)$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

3.2 Electromagnetic Waves

Recall: Ch2, Wave Motion

$$\nabla^2 \Psi = \frac{1}{v} \frac{\partial^2 \Psi}{\partial t^2}$$

$$\boxed{\begin{aligned} \nabla^2 \vec{E} &= \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \\ \nabla^2 \vec{B} &= \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} \end{aligned}}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3 \times 10^8 \, m/s$$

Maxwell: Light is EM wave!

3.2.1 Transverse Wave

$$\vec{E} = \hat{i}E_{ox}e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_x)} + \hat{j}E_{oy}e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_y)} + \hat{k}E_{oz}e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_z)}$$

$$0 = \nabla\cdot\vec{E} = \nabla\cdot\left[\hat{i}E_{ox}e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_x)} + \hat{j}E_{oy}e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_y)} + \hat{k}E_{oz}e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_z)}\right]$$

$$0 = E_{ox}\frac{\partial}{\partial x}e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_x)} + E_{oy}\frac{\partial}{\partial y}e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_y)} + E_{oz}\frac{\partial}{\partial z}e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_z)}$$

$$\vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z$$

$$0 = ik_xE_{ox}e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_x)} + ik_yE_{oy}e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_y)} + ik_zE_{oz}e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_z)} = i\vec{k}\cdot\vec{E}$$

$\boxed{\vec{k}\cdot\vec{E} = 0 \quad \text{Transverse wave} \quad \vec{k}\perp\vec{E}}$

$\boxed{\vec{k}\cdot\vec{B} = 0 \qquad \vec{k}\perp\vec{B}}$

EM Wave: Transverse Wave

Recall: Ch2, Wave Motion

$$\vec{E} = \hat{i}E_{ox}e^{i(kz-\omega t+\phi_{Ex})} + \hat{j}E_{oy}e^{i(kz-\omega t+\phi_{Ey})}$$

$$\vec{B} = \hat{i}B_{ox}e^{i(kz-\omega t+\phi_{Bx})} + \hat{j}B_{oy}e^{i(kz-\omega t+\phi_{By})}$$

$$c = \frac{\omega}{k}$$

$$\boxed{\nabla \Leftrightarrow i\vec{k} \qquad \frac{\partial}{\partial t} \Leftrightarrow -i\omega}$$

$$k = \frac{\omega}{c}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{k} \times \vec{E} = \omega \vec{B}$$

$$E = CB$$

$$\boxed{\vec{B} = \frac{1}{\omega}\vec{k} \times \vec{E} = \frac{1}{c}\frac{c}{\omega}\vec{k} \times \vec{E} = \frac{1}{c}\hat{s} \times \vec{E}}$$

$$\hat{s} = \frac{c}{\omega}\vec{k} = \frac{\vec{k}}{k}$$

EM Wave: Transverse Wave

$$\vec{k} \cdot \vec{E} = 0 \qquad \vec{k} \cdot \vec{B} = 0 \qquad \vec{B} = \frac{1}{\omega}\vec{k} \times \vec{E} = \frac{1}{c}\hat{s} \times \vec{E}$$

$$E = CB$$