PHYS 3038 Optics
L20 Diffraction
Reading Material: Ch10.3.3-10.3.11

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2015, the Year of Light

10.3.3 Circular Apertures

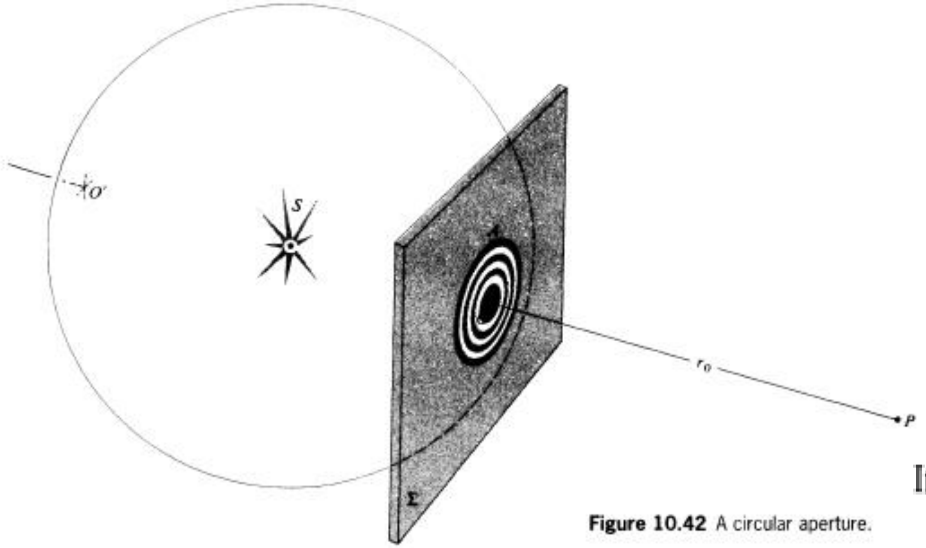


Figure 10.42 A circular aperture.

$$K(\theta) = \tfrac{1}{2}(1 + \cos\theta)$$

For a small hole, θ is small, K varies little

If m is even, then since $K_m \neq 0$,

$$E = (|E_1| - |E_2|) + (|E_3| - |E_4|) + \cdots + (|E_{m-1}| - |E_m|)$$

Because each adjacent contribution is nearly equal,

$$E \approx 0$$

and $I \approx 0$. If, on the other hand, m is odd,

$$E = |E_1| - (|E_2| - |E_3|)$$
$$- (|E_4| - |E_5|) - \cdots - (|E_{m-1}| - |E_m|)$$

and $$E \approx |E_1|$$

which is roughly twice the amplitude of the unobstructed wave. This is truly an amazing result. By inserting a screen in the path of the wave, thereby blocking out most of the wavefront, we have increased the irradiance at P by a factor of four.

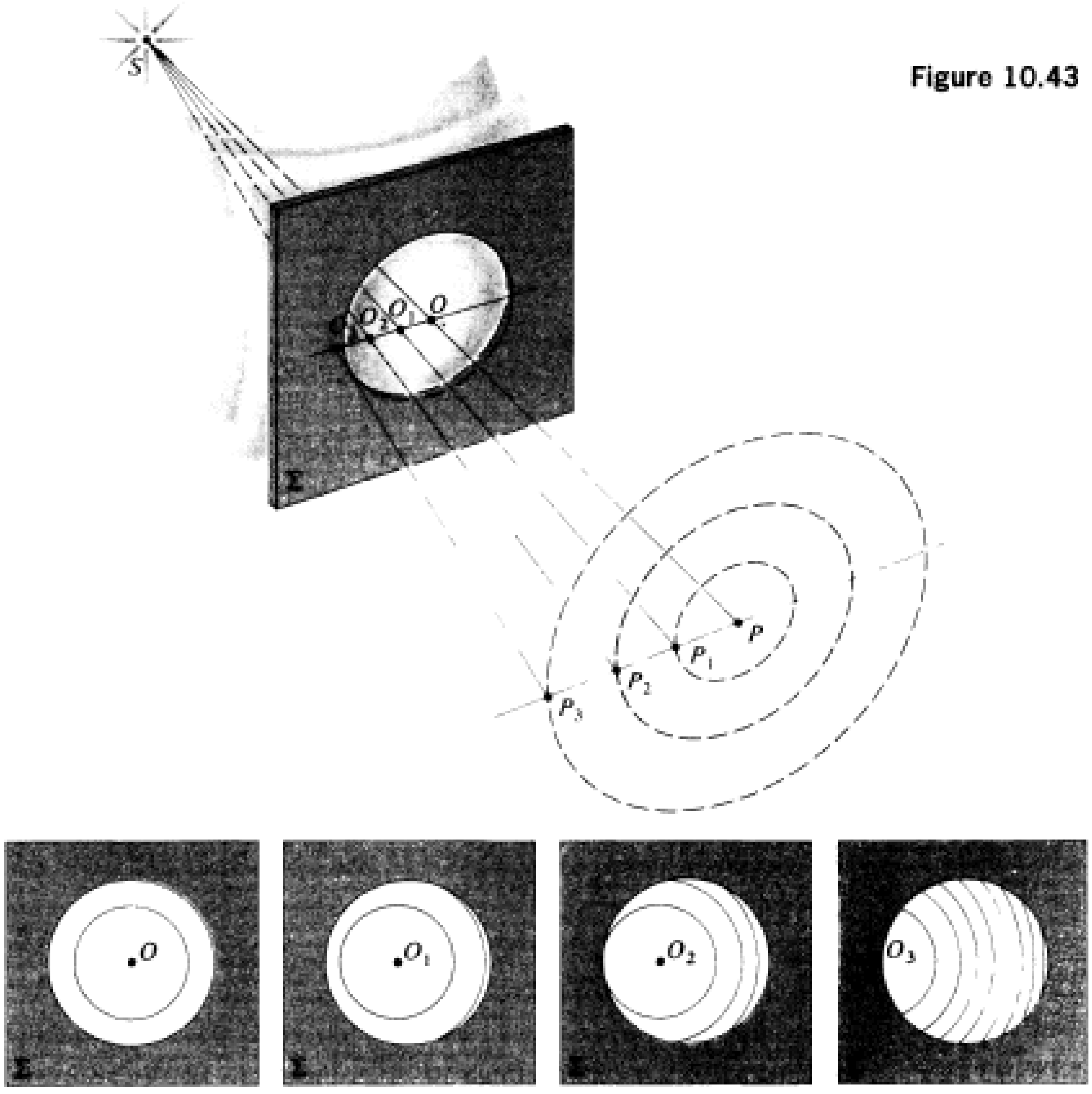
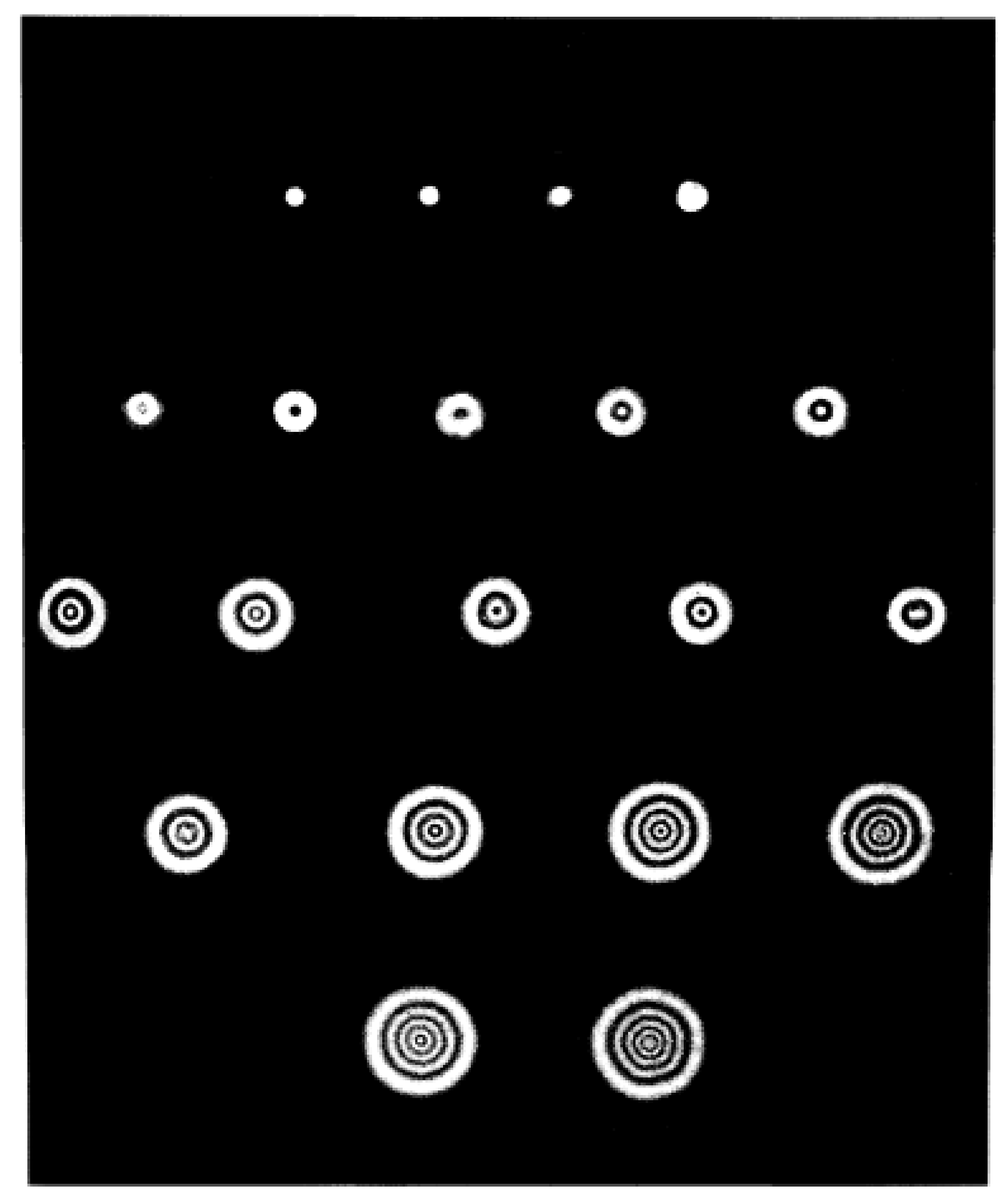
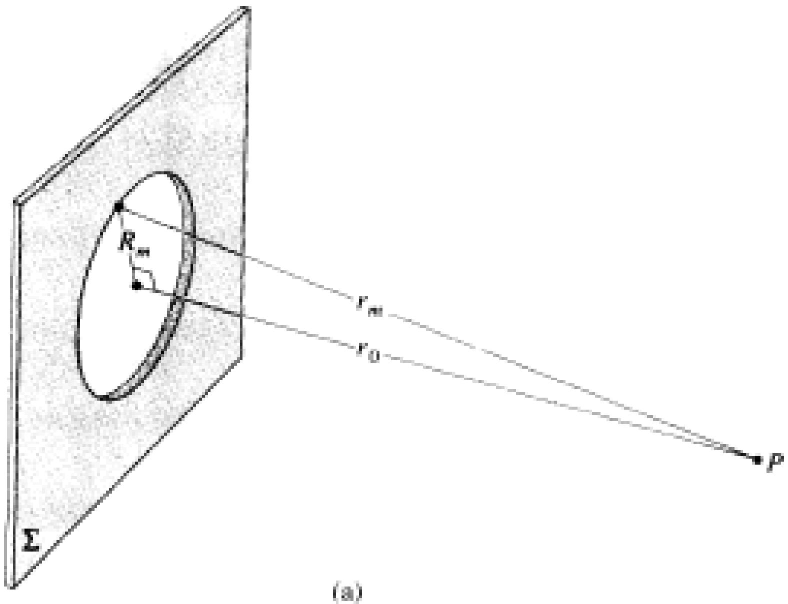


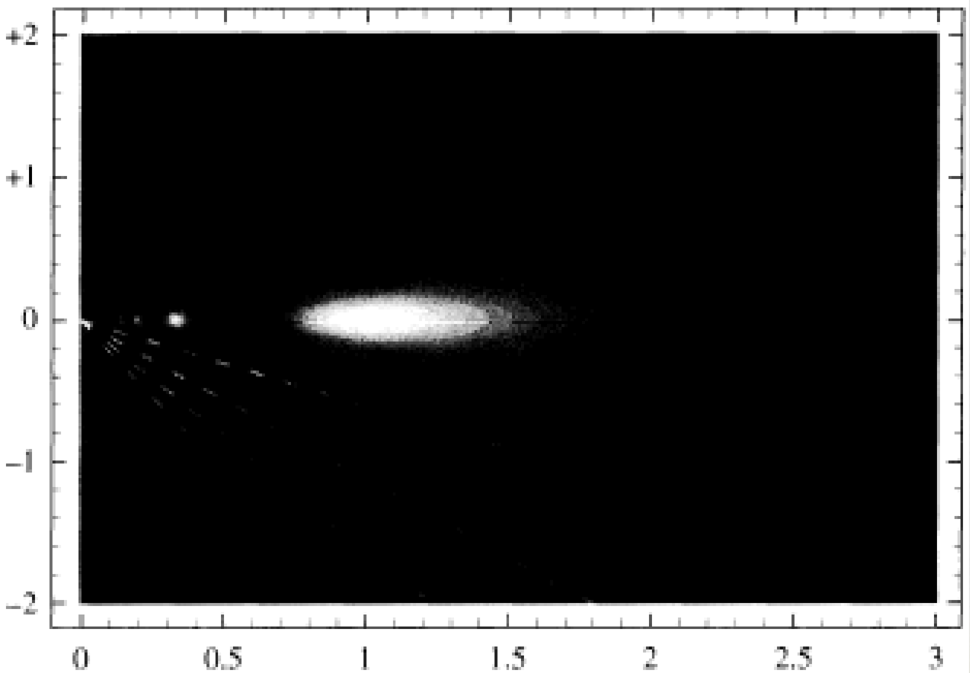
Figure 10.43 Zones in a circular aperture.

Diffraction Patterns for Circular Apertures





(a)



(b)

Plane Wave – Circular Hole

$$R_m^2 = (r_0 + m\lambda/2)^2 - r_0^2$$

$$R_m^2 = mr_0\lambda + m^2\lambda^2/4$$

Under most circumstances, the second term in Eq. (10.90) is negligible as long as m is not extremely large; consequently

$$R_m^2 = mr_0\lambda$$

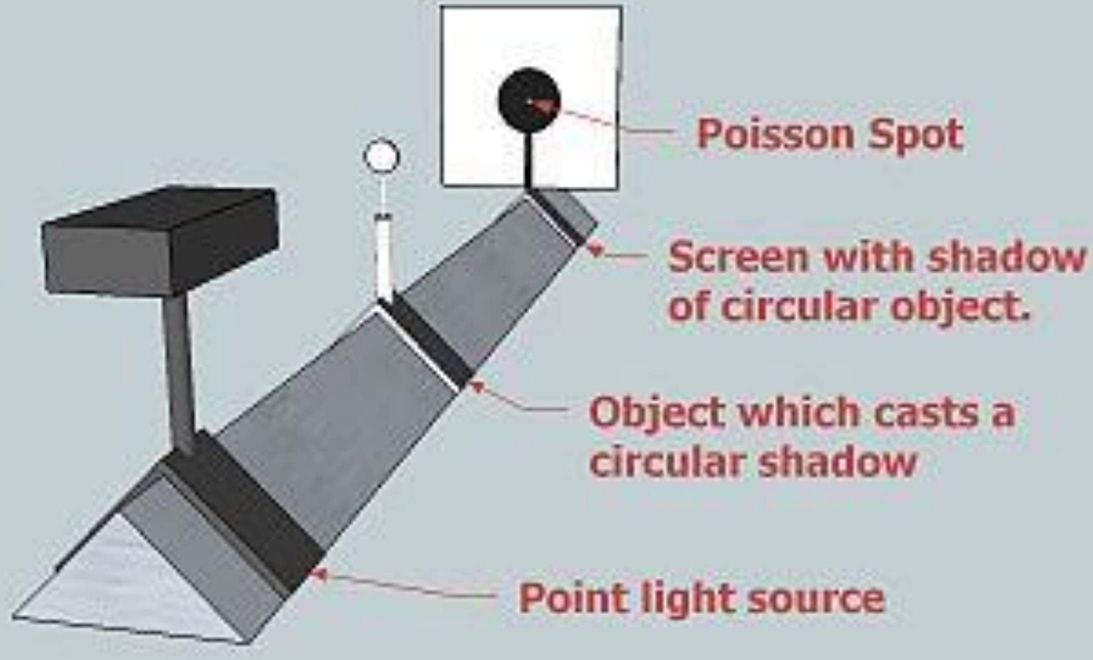
10.3.4 Circular Obstacles

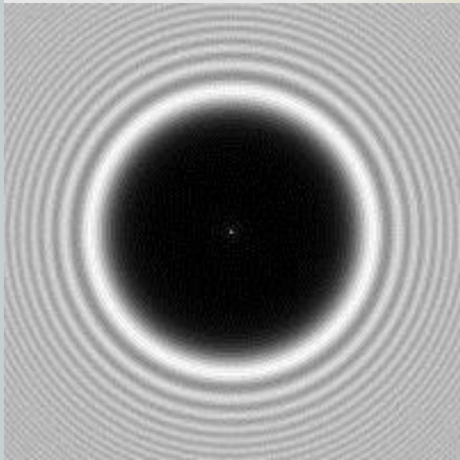
Augustin-Jean Fresnel

Poisson's Spot

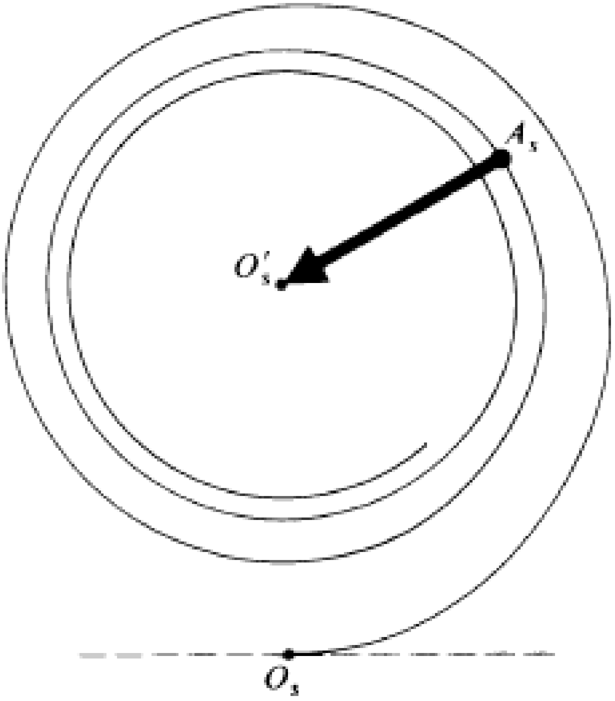
Siméon Poisson

https://en.wikipedia.org/wiki/Arago_spot





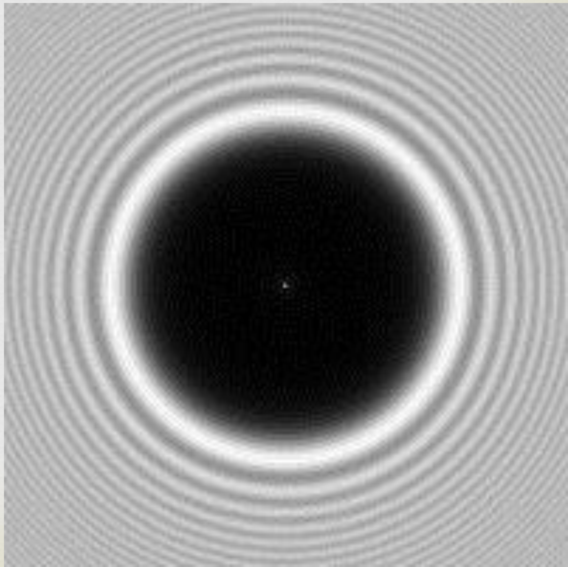
Circular Obstacles



$$E = |E_{\ell+1}| - |E_{\ell+2}| + \cdots + |E_m|$$

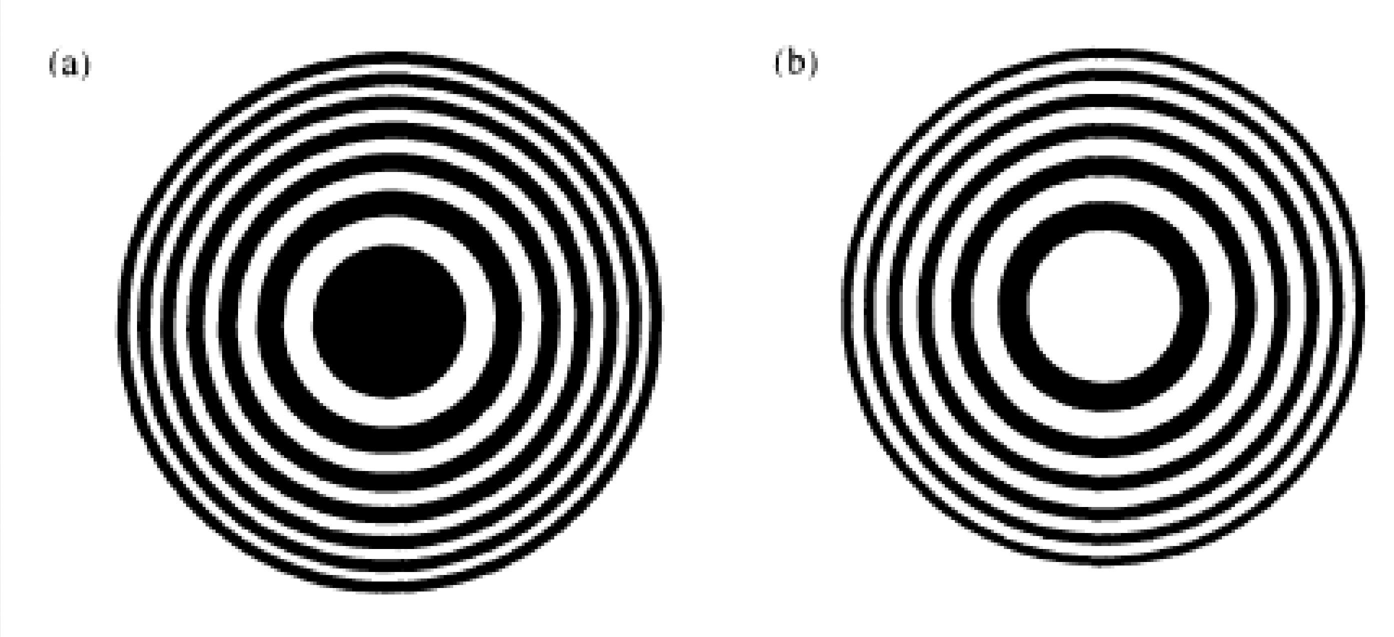
(where, as before, there is no absolute significance to the signs other than that alternate terms must subtract). Unlike the analysis for the circular aperture, E_m now approaches zero, because $K_m \to 0$. The series must be evaluated in the same manner as that of the unobstructed wave [Eqs. (10.78) and (10.79)]. Repeating that procedure yields

$$E \approx \frac{|E_{\ell+1}|}{2} \tag{10.92}$$

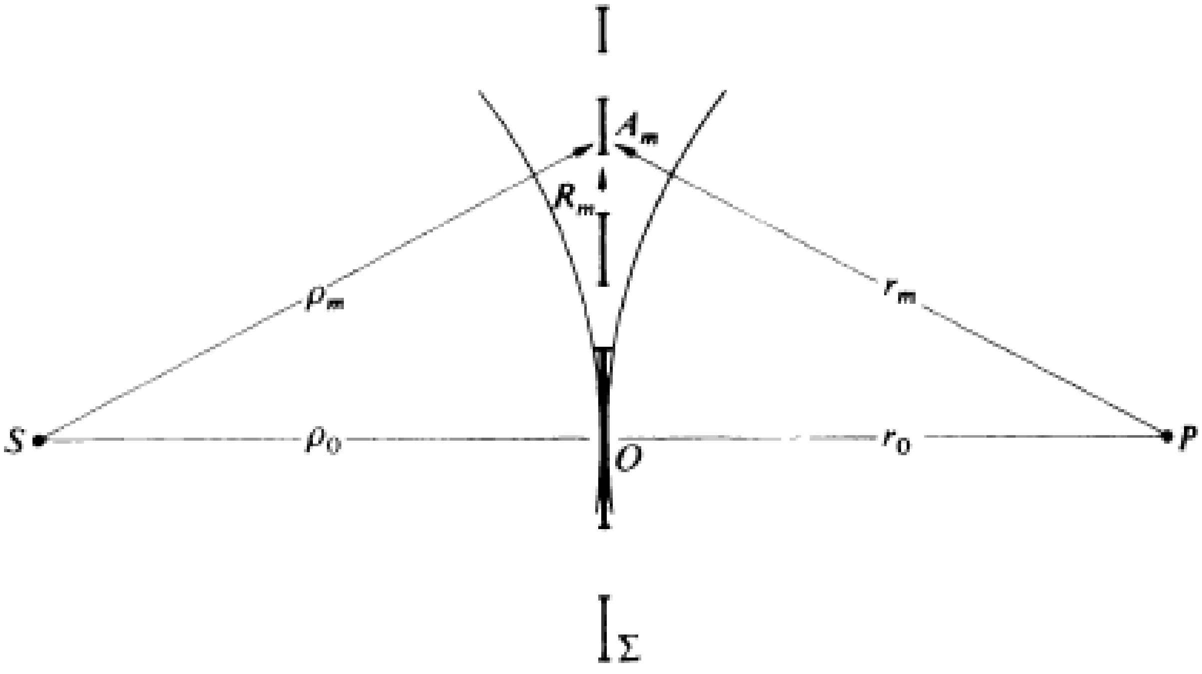


Fresnel Zone Plate

$$E = E_1 + E_3 + E_5 + \cdots + E_{39} \approx 20E_1.$$



Fresnel Zone Plate



$$(\rho_m + r_m) - (\rho_0 + r_0) = m\lambda/2$$

Clearly, $\rho_m = (R_m^2 + \rho_0^2)^{1/2}$ and $r_m = (R_m^2 + r_0^2)^{1/2}$. Expand both these expressions using the binomial series. Since R_m is comparatively small, retaining only the first two terms yields

$$\rho_m = \rho_0 + \frac{R_m^2}{2\rho_0} \quad \text{and} \quad r_m = r_0 + \frac{R_m^2}{2r_0}$$

Finally, substituting into Eq. (10.93), we obtain

$$\left(\frac{1}{\rho_0} + \frac{1}{r_0}\right) = \frac{m\lambda}{R_m^2} \qquad (10.94)$$

$$f_1 = \frac{R_m^2}{m\lambda}$$

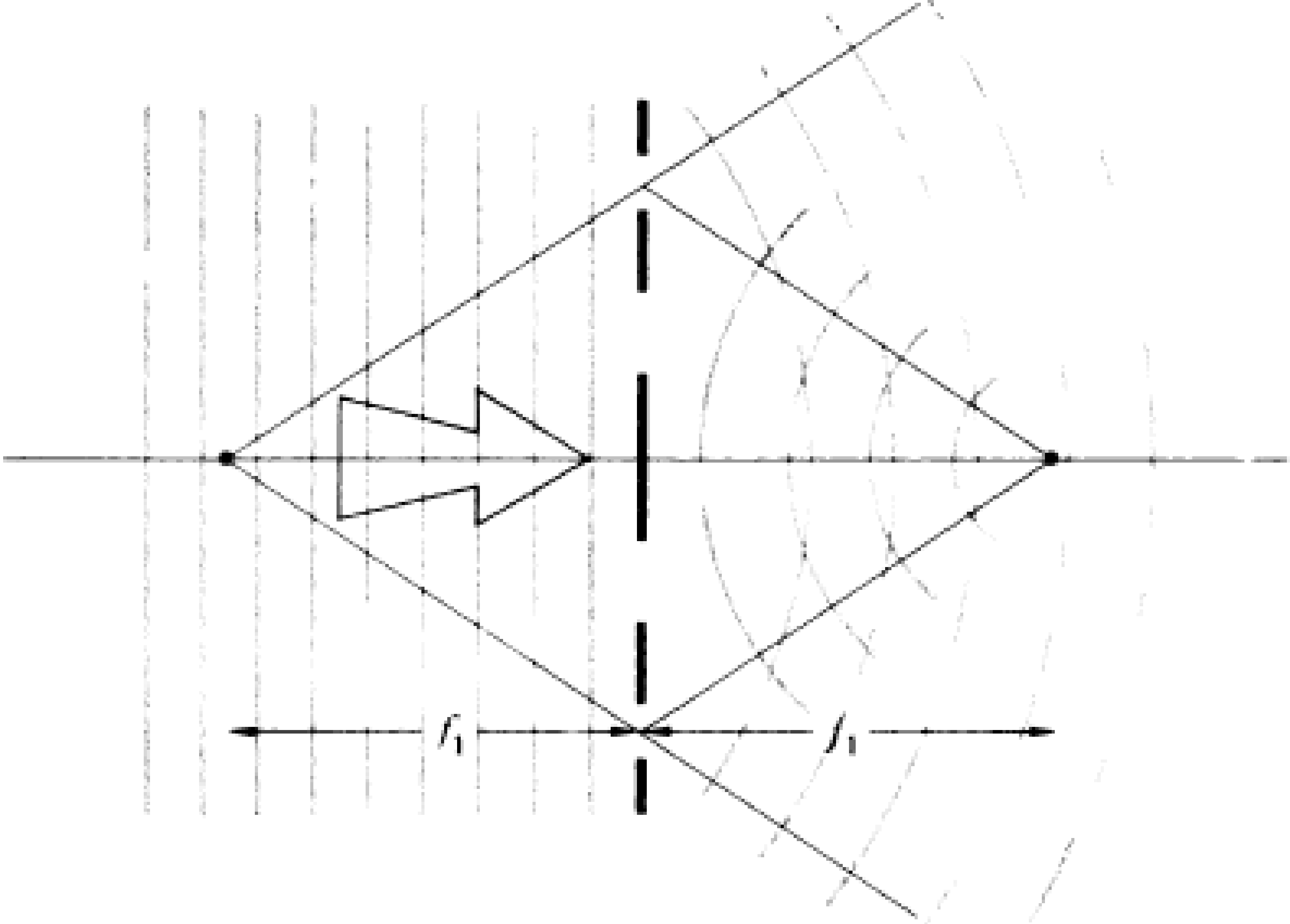
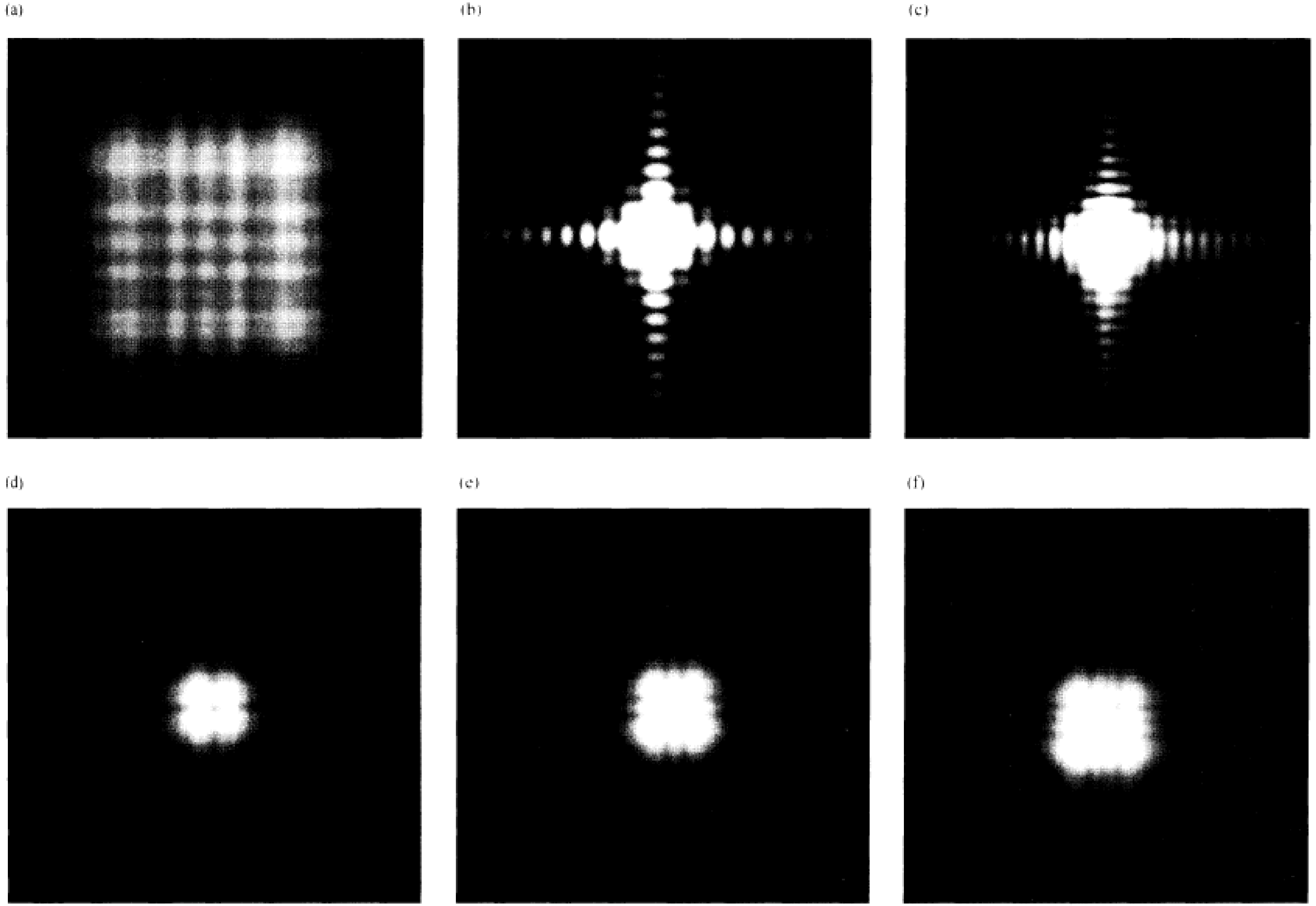


Figure 10.48 Zone-plate foci.

Rectangular Aperture



(*a*) A typical Fresnel pattern for a square aperture. (*b*)–(*f*) A series of Fresnel patterns for increasing square apertures under identical conditions. Note that as the hole gets larger, the pattern changes from a spread-out Fraunhofer-like distribution to a far more localized structure. (Photos by E. H.)