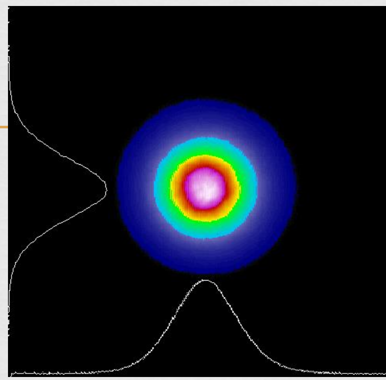
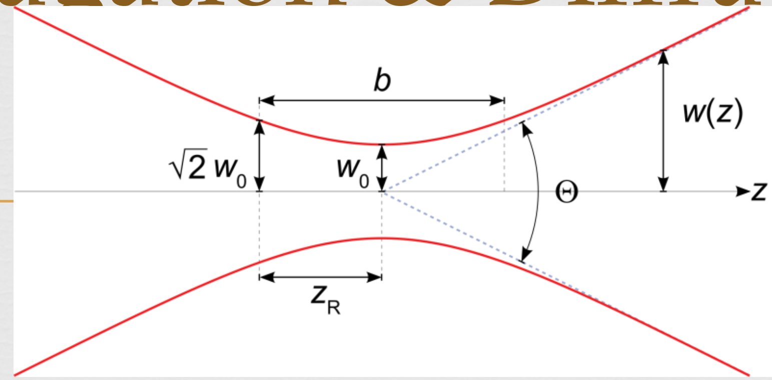
PHYS 3038 Optics
L19 Diffraction
Reading Material: Ch10.2.7-10.3.2

Shengwang Du

2015, the Year of Light

Beam Propagation & Diffraction





Rayleigh range: $Z_R = \frac{pD_0^2}{4l}$

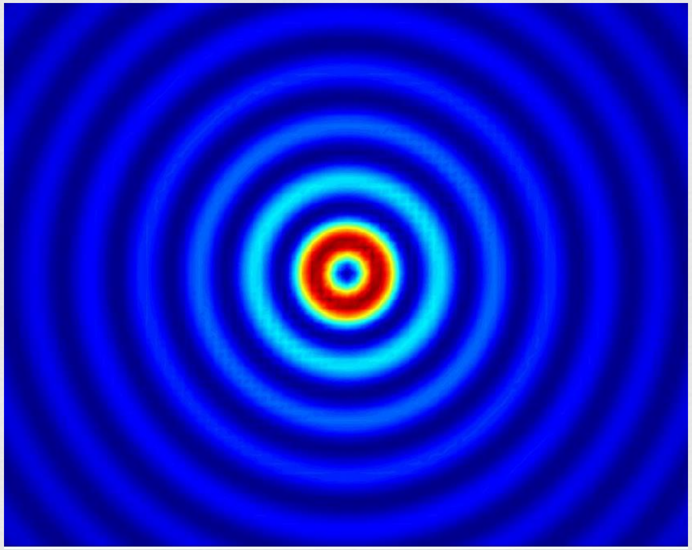
Nondiffracting beam: the wave front pattern does change over propagation.

- Plane wave
- Any others?

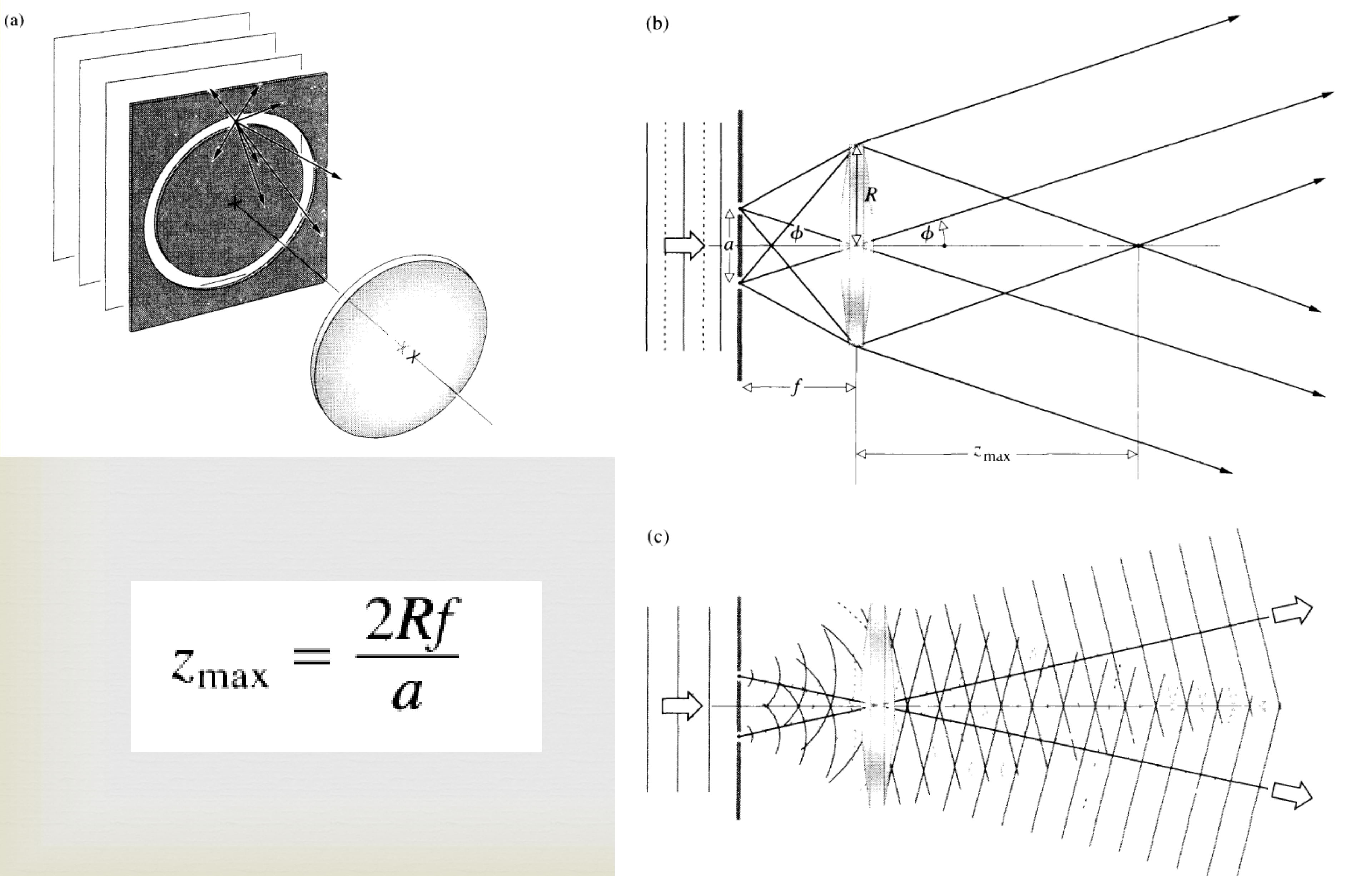
$$E(x, y, z, t) = A(x, y)e^{i(k_0 z - wt)}$$

The Zero-Order Bessel Beam

$$\tilde{E}(r, \theta, z, t) \propto J_0(k_\perp r) e^{i(k_\parallel z - \omega t)}$$



(Quasi) Bessel Beam Generation



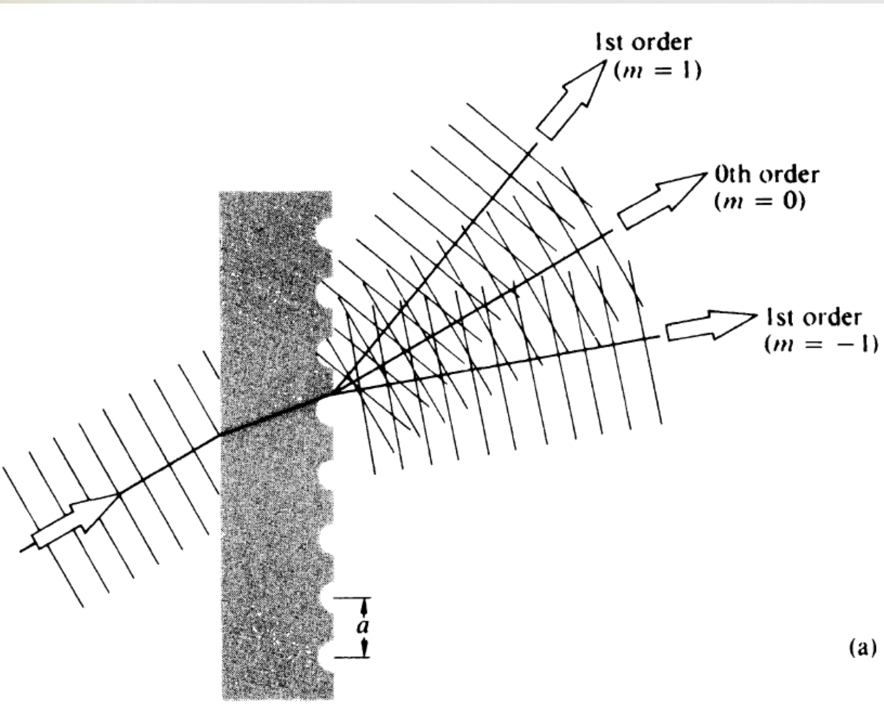
$$z_{max} = \frac{2Rf}{a}$$

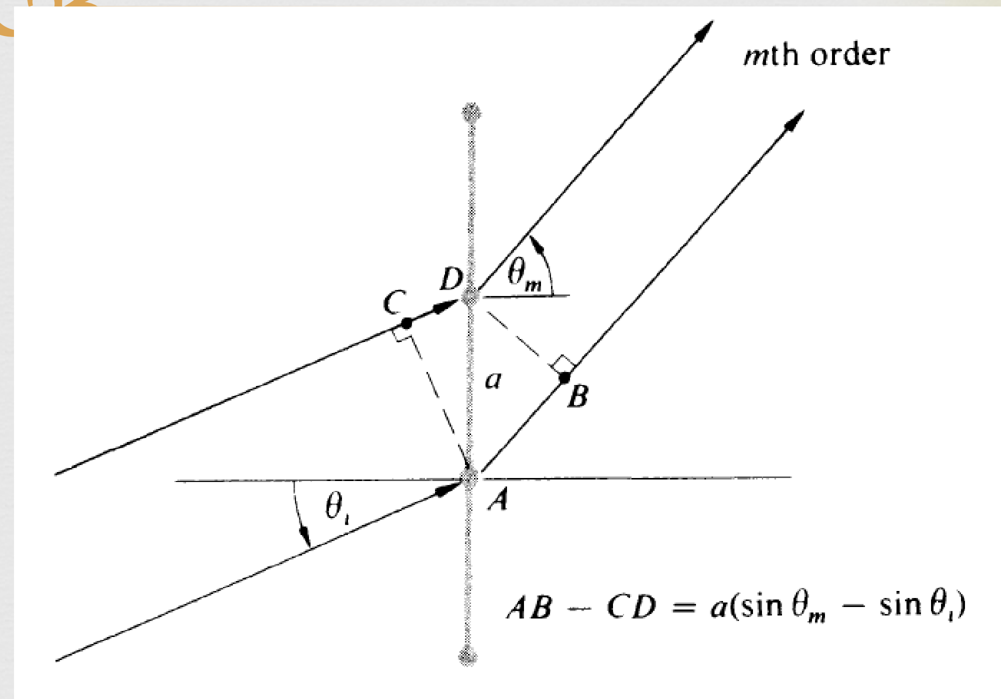
10.2.8 Diffraction Grating

A repetitive array of diffracting elements, either apertures or obstacles, that has the effect of producing periodic alterations in the phase, amplitude, or both of an emergent wave is said to be a **diffraction grating**. One of the simplest such arrange-

- Transmission grating
  - Transmission amplitude grating
  - Transmission phase grating
- Reflection grating

Transmission Grating

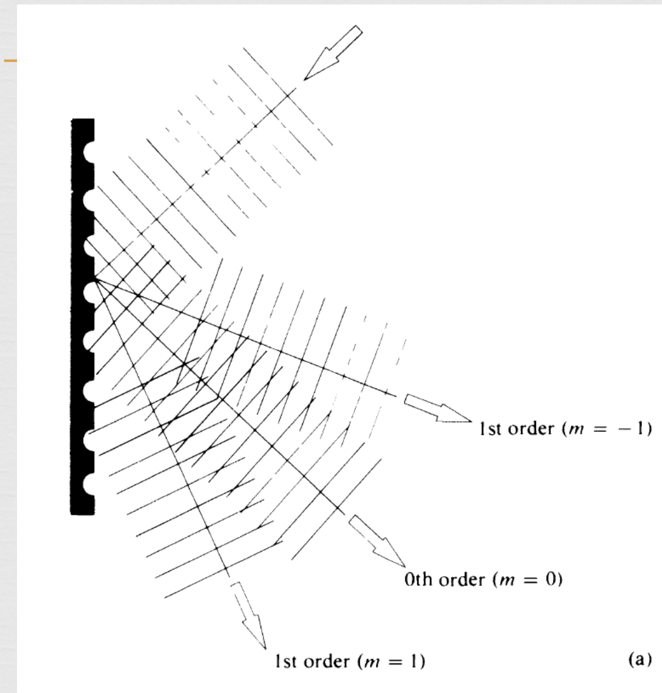


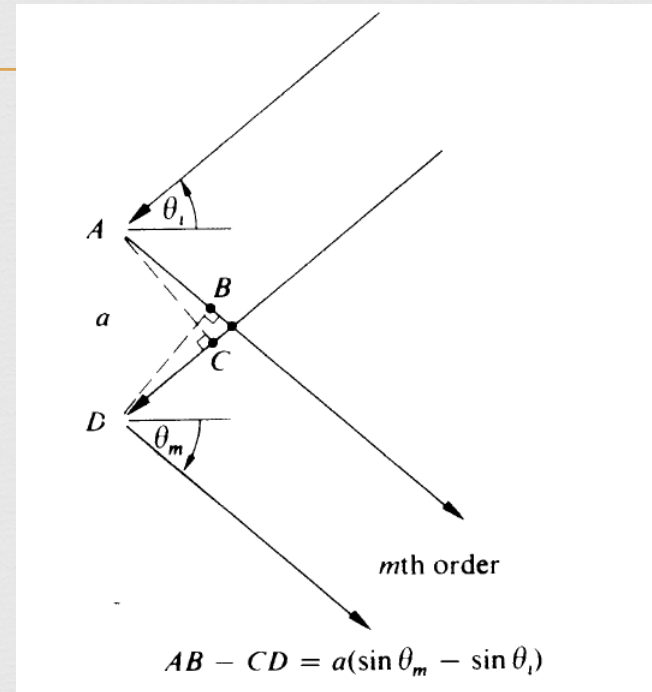


Grating equation $a(\sin\theta_m - \sin\theta_i) = m\lambda$

For perpendicular incident: $a\sin\theta_m = m\lambda$

Reflection Grating





Grating equation $$a(\sin q_m - \sin q_i) = ml$$

For perpendicular incident: $$a \sin q_m = ml$$

Blazed Grating

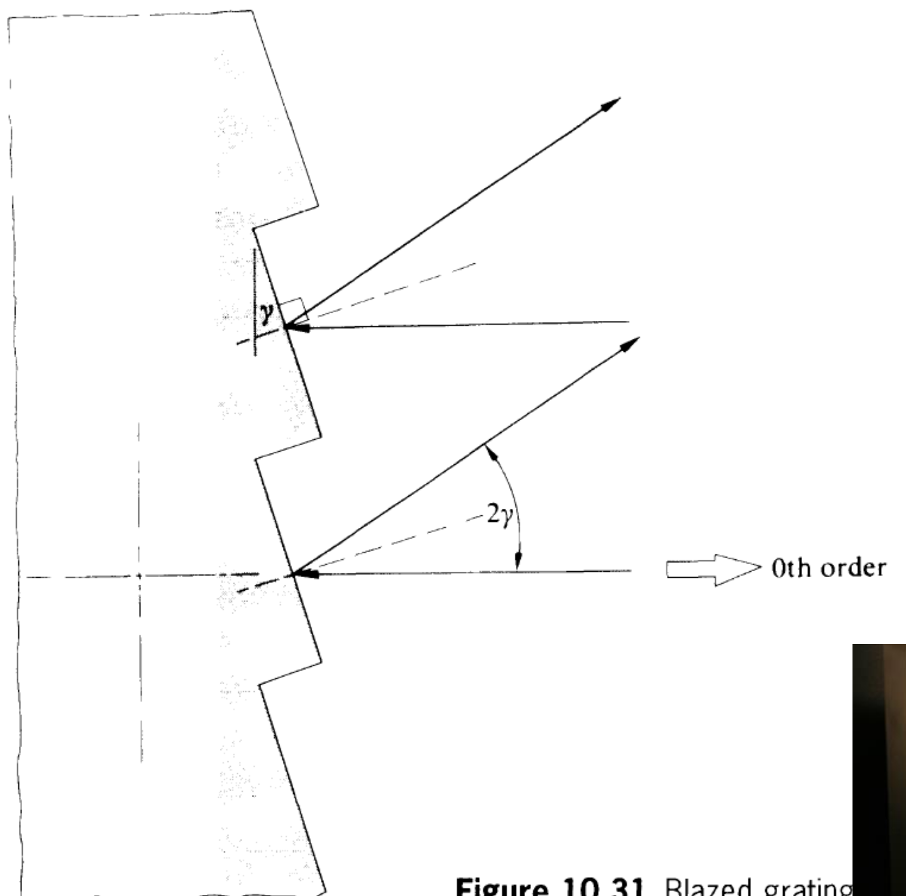
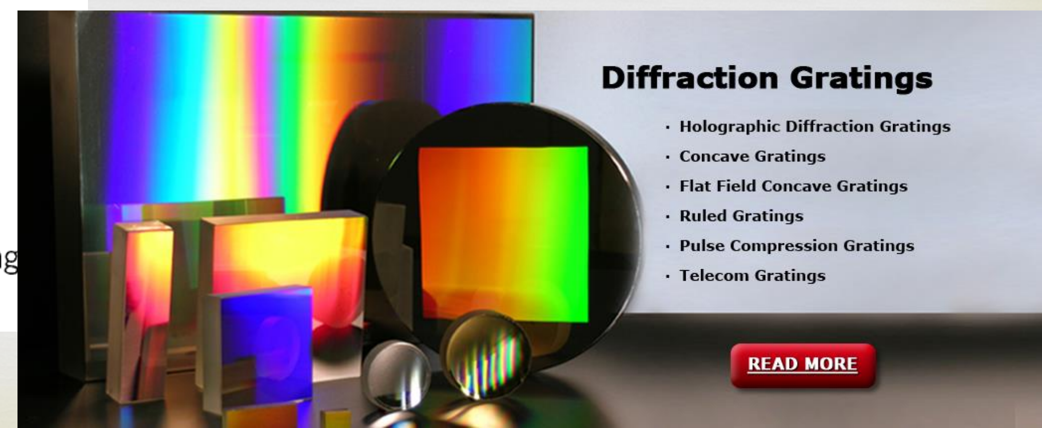


Figure 10.31 Blazed grating

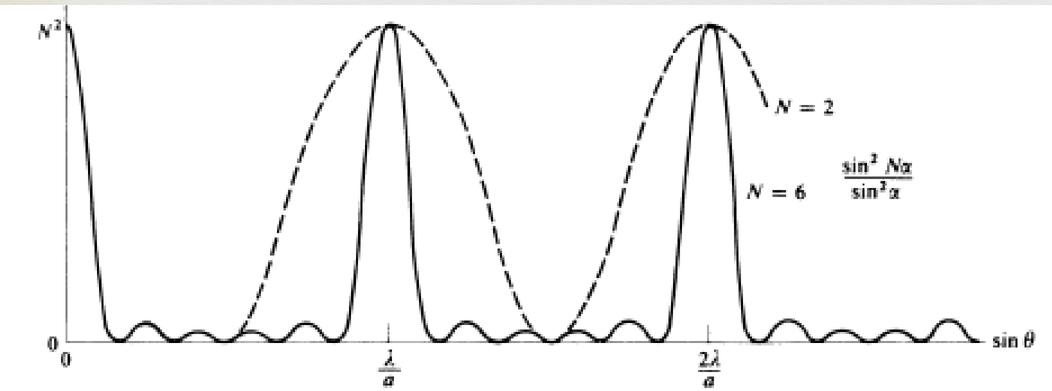


Grating Spectroscopy

Principal maxima

$$a(\sin\theta_m - \sin\theta_i) = m\lambda$$

$$\alpha = (ka/2)(\sin\theta_m - \sin\theta_i) = m\pi$$



Angular width

$$\Delta\alpha = (ka/2)\cos\theta\,(\Delta\theta) = 2\pi/N \qquad (10.62)$$

$$\Delta\theta = 2\lambda/(Na\cos\theta_m) \qquad (10.63)$$

$$(\Delta\theta)_{\min} = \lambda/(Na\cos\theta_m)$$

Angular dispersion

$$\mathcal{D} \equiv d\theta/d\lambda$$

$$a(\sin\theta_m - \sin\theta_i) = m\lambda$$

$$\mathcal{D} = m/(a\cos\theta_m)$$

Grating Spectroscopy

Resolution (angular separation) $(\Delta\theta)_{\min} = \lambda/(Na \cos\theta_m)$

Angular dispersion $\mathscr{D} \equiv d\theta/d\lambda$ $\mathscr{D} = m/(a \cos\theta_m)$

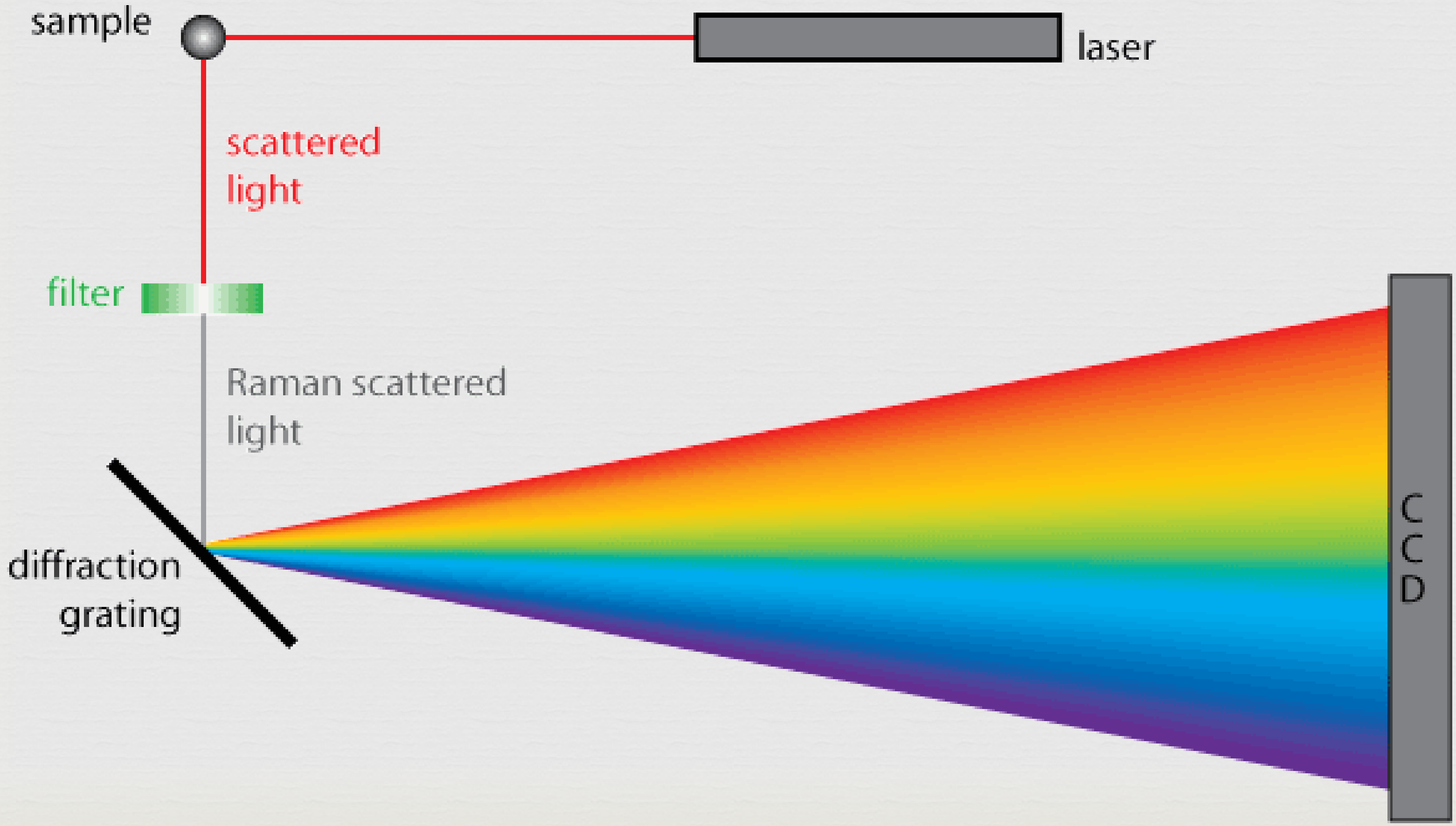
$$\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{a\cos\theta_m} \qquad \Delta\theta = \frac{m\Delta\lambda}{a\cos\theta_m}$$

To resolve different wavelength $\Delta\theta = \dfrac{m\Delta\lambda}{a\cos\theta_m} \geq (\Delta\theta)_{\min} = \dfrac{\lambda}{Na\cos\theta_m}$

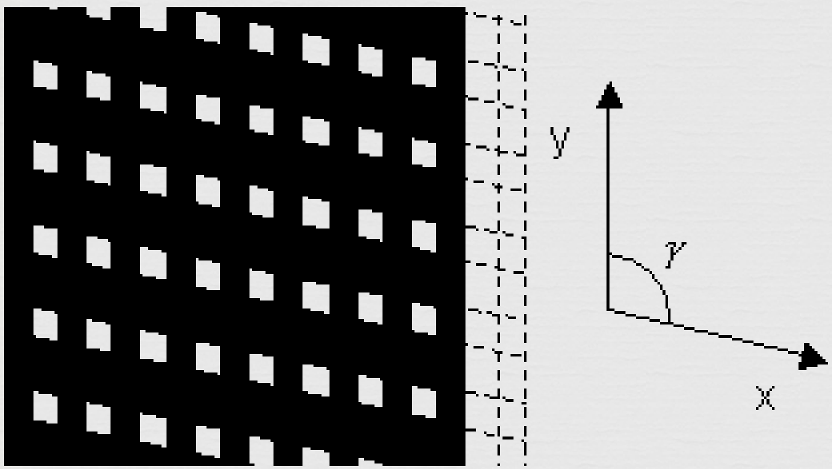
Wavelength Resolution:

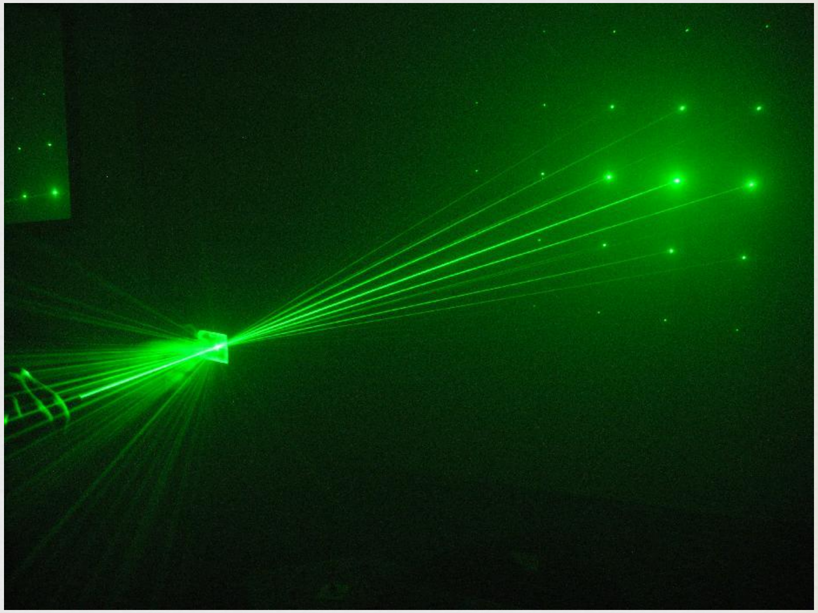
$$(\Delta\lambda)_{\min} \geq \frac{\lambda}{Nm}$$

Grating Spectroscopy

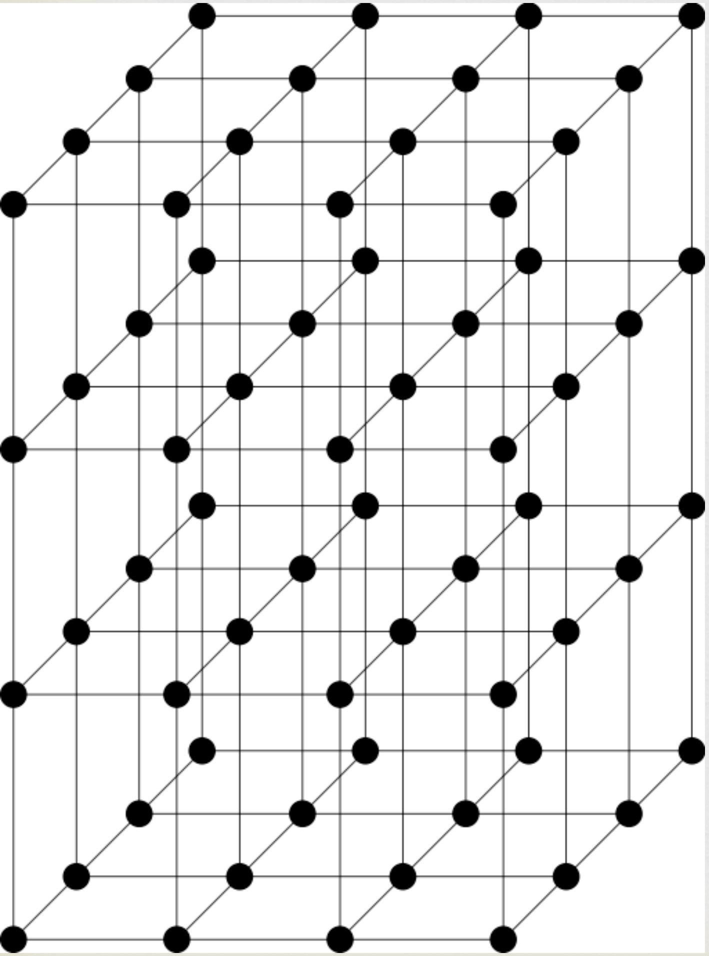


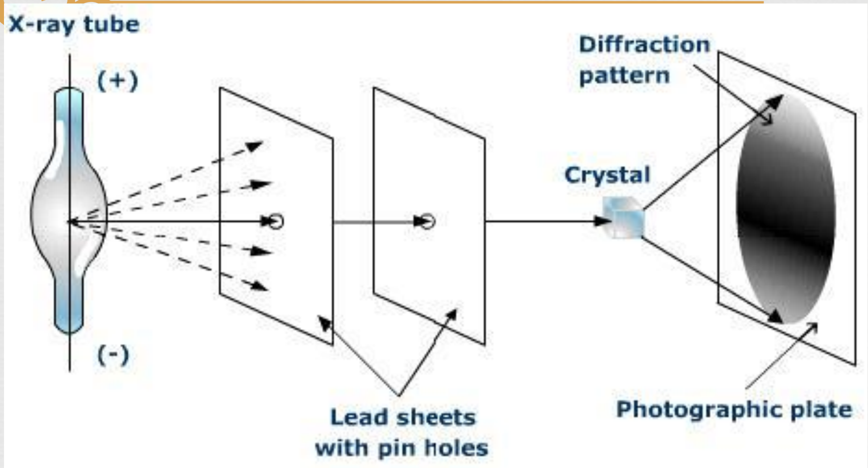
2D Gratings

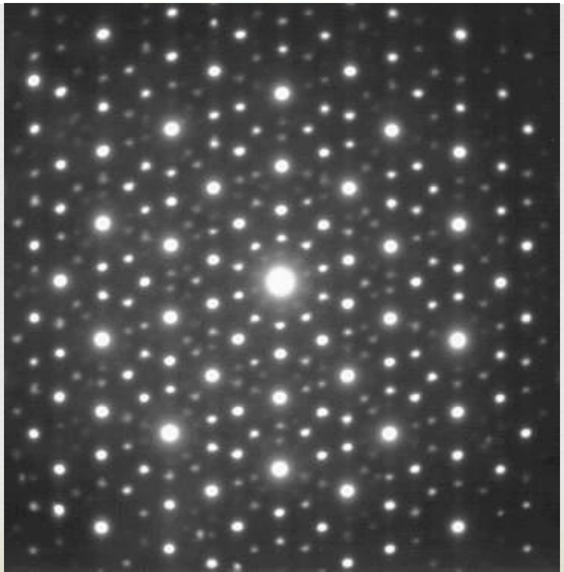




3D Grating : Crystal







10.3 Fresnel Diffraction

Distance from the aperture: R
Aperture size: a
Wavelength: λ

- Fraunhofer (far-field) diffraction:

- Fresnel (near-field) diffraction:

Obliquity of Secondary Wavelets

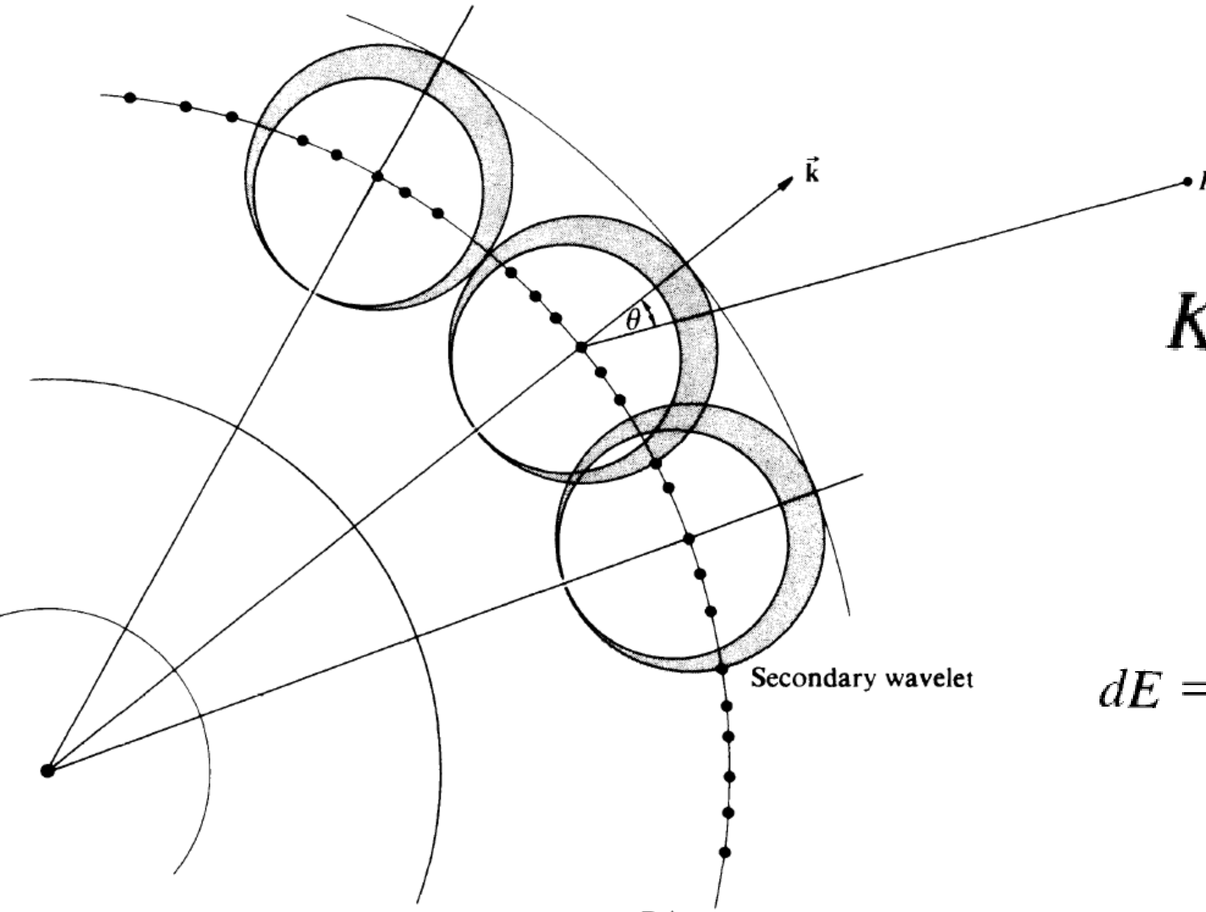


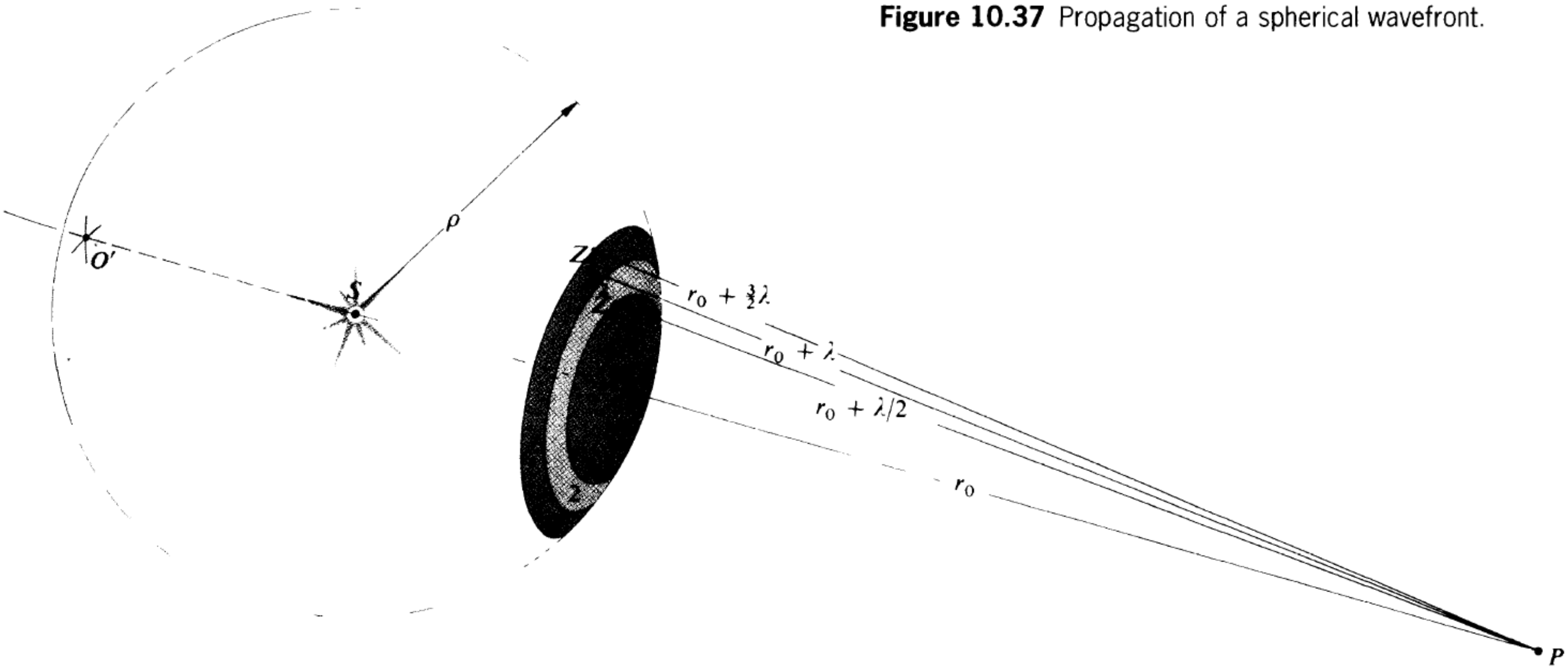
Figure 10.36 Secondary wavelets.

$$K(\theta) = \tfrac{1}{2}(1 + \cos\theta)$$

$$dE = K\frac{\mathcal{E}_A}{r}\cos\left[\omega t - k(\rho + r)\right] dS$$

Propagation of a Spherical Wavefront

Figure 10.37 Propagation of a spherical wavefront.



Fresnel Zones (half-period zones)

Propagation of a Spherical Wavefront

Fresnel Zones (half-period zones)

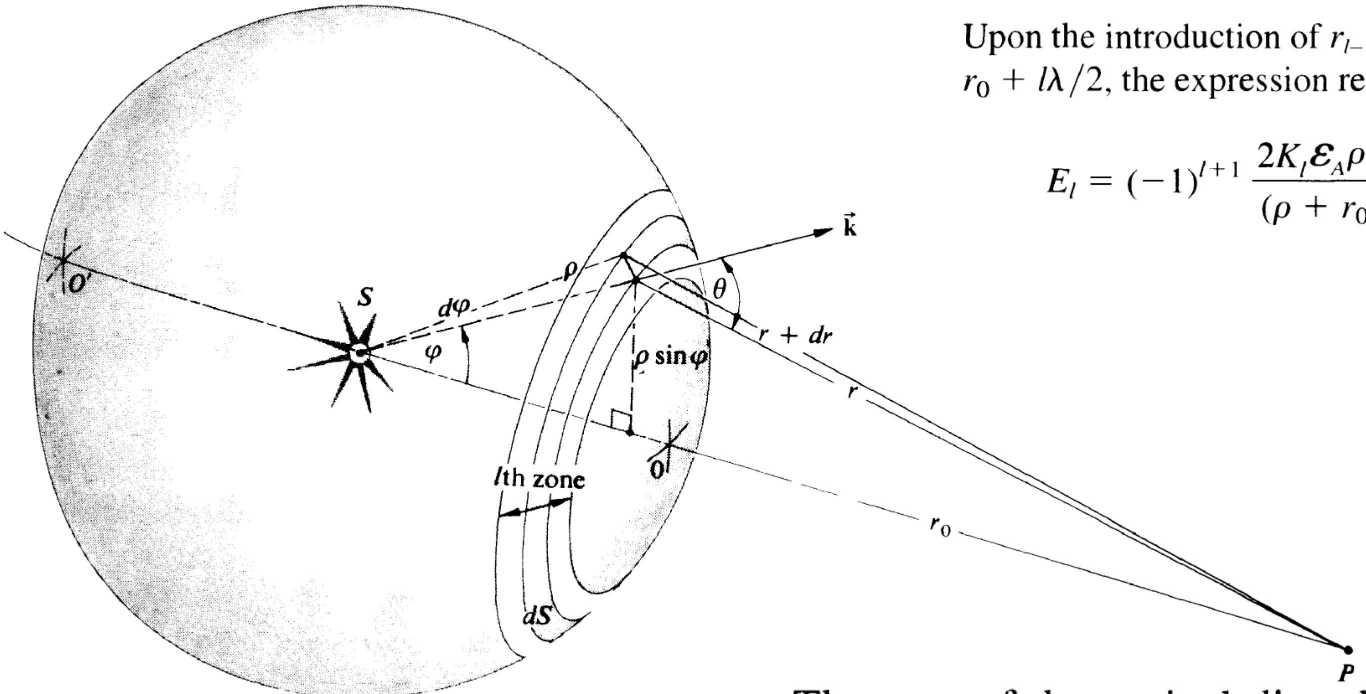


Figure 10.38 Propagation of a spherical wavefront.

Upon the introduction of $r_{l-1} = r_0 + (l-1)\lambda/2$ and $r_l = r_0 + l\lambda/2$, the expression reduces (Problem 10.42) to

$$E_l = (-1)^{l+1} \frac{2K_l \mathcal{E}_A \rho\lambda}{(\rho + r_0)} \sin [\omega t - k(\rho + r_0)] \qquad (10.76)$$

The sum of the optical disturbances from all m zones at P is

$$E = E_1 + E_2 + E_3 + \cdots + E_m$$

and since these alternate in sign, we can write

$$E = |E_1| - |E_2| + |E_3| - \cdots \pm |E_m| \qquad (10.77)$$

Propagation of a Spherical Wavefront

If m is odd, the series can be reformulated in two ways, either as

$$E = \frac{|E_1|}{2} + \left(\frac{|E_1|}{2} - |E_2| + \frac{|E_3|}{2}\right) + \left(\frac{|E_3|}{2} - |E_4| + \frac{|E_5|}{2}\right) + \cdots + \left(\frac{|E_{m-2}|}{2} - |E_{m-1}| + \frac{|E_m|}{2}\right) + \frac{|E_m|}{2} \tag{10.78}$$

or as

$$E = |E_1| - \frac{|E_2|}{2} - \left(\frac{|E_2|}{2} - |E_3| + \frac{|E_4|}{2}\right) - \left(\frac{|E_4|}{2} - |E_5| + \frac{|E_6|}{2}\right) + \cdots + \left(\frac{|E_{m-3}|}{2} - |E_{m-2}| + \frac{|E_{m-1}|}{2}\right) - \frac{|E_{m-1}|}{2} + |E_m|$$

There are now two possibilities: either $|E_l|$ is greater than the arithmetic mean of its two neighbors $|E_{l-1}|$ and $|E_{l+1}|$, or it is less than that mean. This is really a question concerning the rate of change of $K(\theta)$. When

$$|E_l| > (|E_{l-1}| + |E_{l+1}|)/2$$

each bracketed term is negative. It follows from Eq. (10.78) that

$$E < \frac{|E_1|}{2} + \frac{|E_m|}{2} \tag{10.80}$$

and from Eq. (10.79) that

$$E > |E_1| - \frac{|E_2|}{2} - \frac{|E_{m-1}|}{2} + |E_m| \tag{10.81}$$

Since the obliquity factor goes from 1 to 0 over a great many zones, we can neglect any variation between adjacent zones, that is, $|E_1| \approx |E_2|$ and $|E_{m-1}| \approx |E_m|$. Expression (10.81), to the same degree of approximation, becomes

$$E > \frac{|E_1|}{2} + \frac{|E_m|}{2} \tag{10.82}$$

We conclude from Eqs. (10.80) and (10.82) that

$$E \approx \frac{|E_1|}{2} + \frac{|E_m|}{2} \tag{10.83}$$

m: odd

This same result is obtained when

$$|E_l| < (|E_{l-1}| + |E_{l+1}|)/2$$

Fresnel Zones

- m: odd

$$E \approx \frac{|E_1|}{2} + \frac{|E_m|}{2}$$

- m: even

$$E \approx \frac{|E_1|}{2} - \frac{|E_m|}{2}$$

For a large m, $K_m \rightarrow 0$,

$$E \approx \frac{|E_1|}{2}$$

Spherical Wave

If the primary wave were simply to propagate from S to P in a time t, it would have the form

$$E = \frac{\mathcal{E}_0}{(\rho + r_0)} \cos\,[\omega t - k(\rho + r_0)] \qquad (10.86)$$

Yet the disturbance synthesized from secondary wavelets, Eqs. (10.76) and (10.85), is

$$E = \frac{K_1 \mathcal{E}_A \rho \lambda}{(\rho + r_0)} \sin\,[\omega t - k(\rho + r_0)] \qquad (10.87)$$

$$K_1 = 1$$

$$\mathcal{E}_A \rho \lambda = \mathcal{E}_0;$$

$$\mathcal{E}_A = E_0(\rho)/\lambda$$

Vibration Curve