PHYS 3038 Optics
L17 Diffraction
Reading Material: Ch10.1-2

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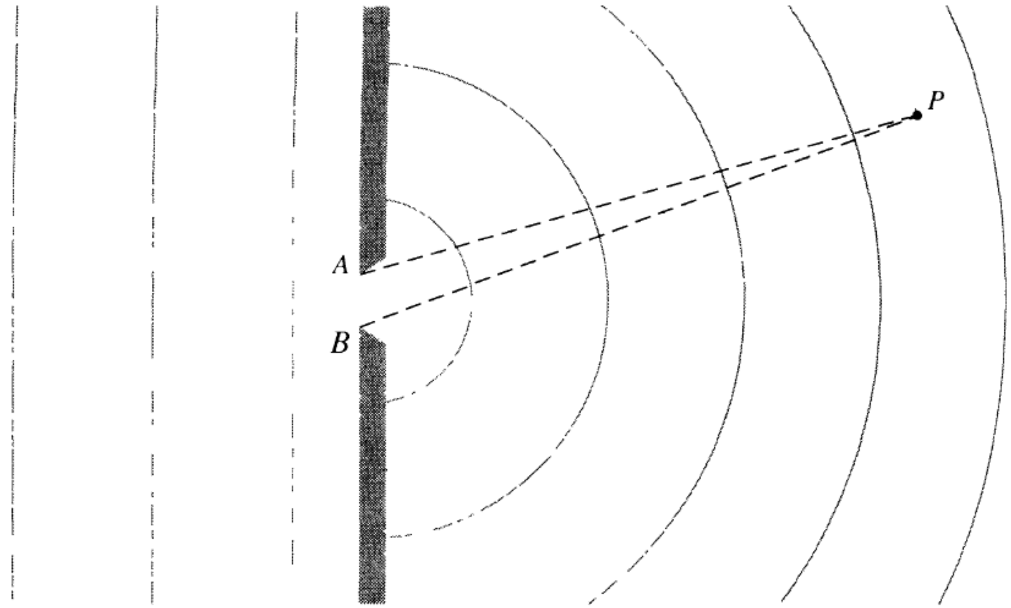
2015, the Year of Light

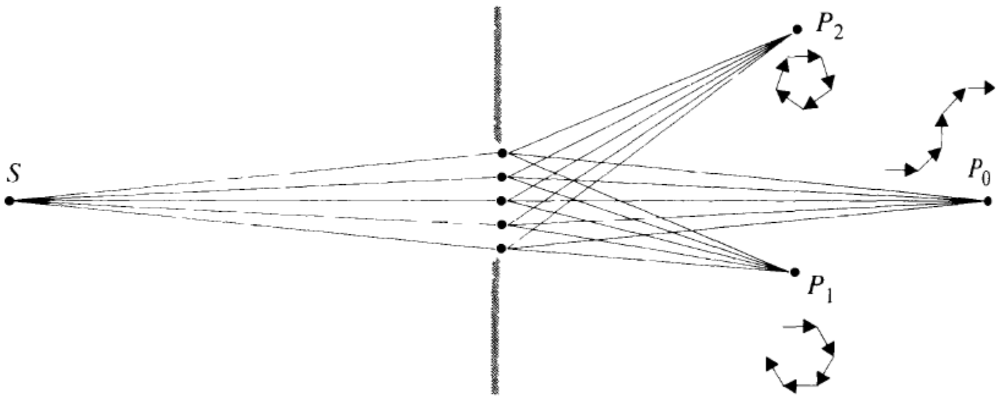
Diffraction: Self Interference

The difficulty was resolved by Fresnel with his addition of the concept of interference. The corresponding **Huygens–Fresnel Principle** states that ***every unobstructed point of a wavefront, at a given instant, serves as a source of spherical secondary wavelets (with the same frequency as that of the primary wave). The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitudes and relative phases).***

Diffraction at a Small Aperature

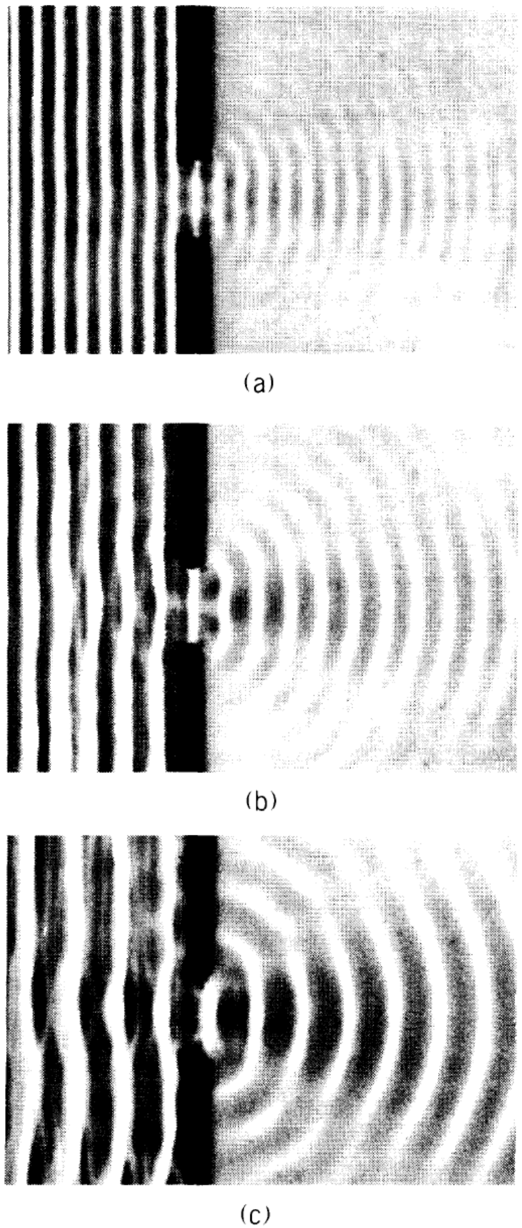


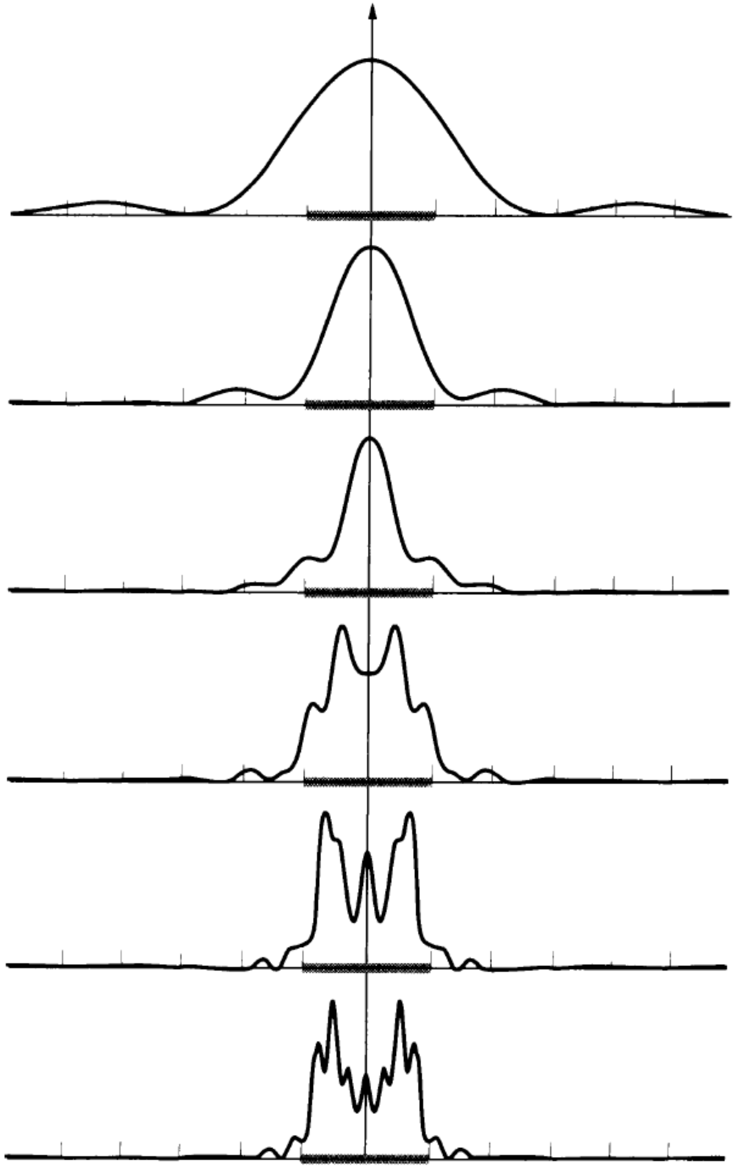
(a)



(b)

Diffraction through an Aperature



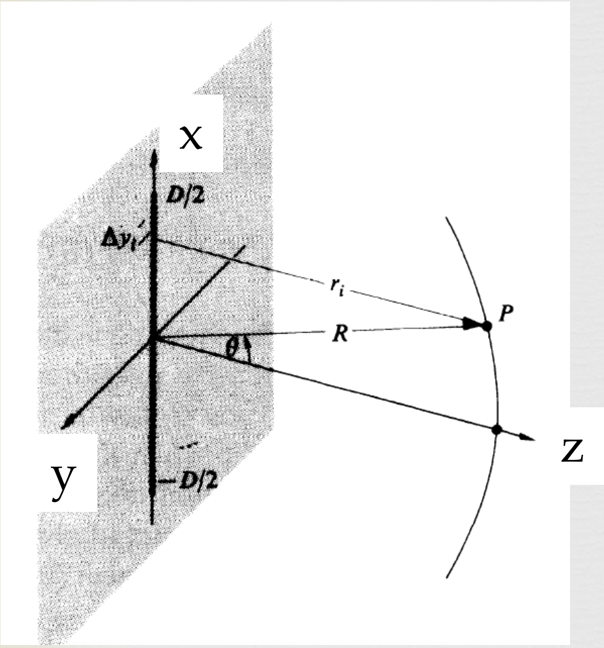


Fraunhofer and Fresnel Diffraction

Distance from the aperture: R
Aperture size: a
Wavelength: λ

- Fraunhofer (far-field) diffraction: $R > \frac{a^2}{\lambda}$
- Fresnel (near-field) diffraction: $R < \frac{a^2}{\lambda}$

Single-Line Aperture



$$E = \int_{-D/2}^{D/2} \frac{\varepsilon_L}{r} e^{i(kr-\omega t)} dx$$

ε_L: Source strength per unit length

Position P: (x_p, y_p, z_p)

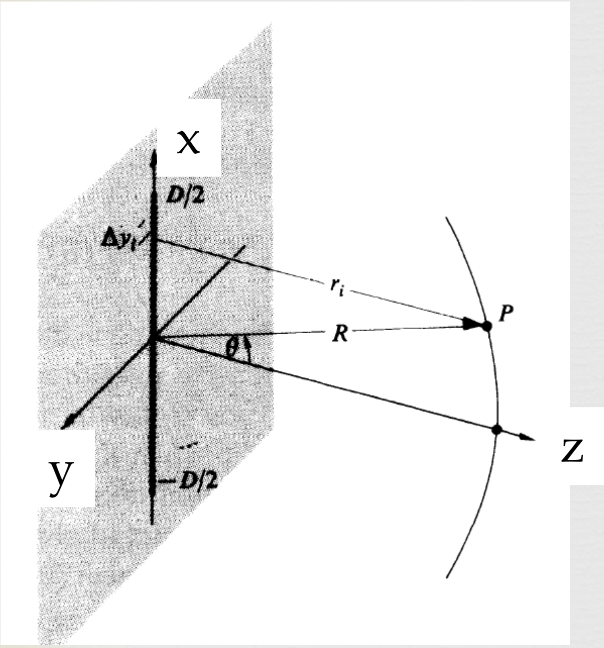
$$r = \sqrt{(x_p - y)^2 + y_p^2 + z_p^2}$$

$$E = e^{-i\omega t} \int_{-D/2}^{D/2} \frac{\varepsilon_L}{\sqrt{(x_p - x)^2 + y_p^2 + z_p^2}} e^{i\frac{2\pi}{\lambda}\sqrt{(x_p-x)^2+y_p^2+z_p^2}} dy$$

Fraunhore Diffraction

R = $\sqrt{x_p^2 + y_p^2 + z_p^2}$ >> D

$$\sin\theta = \frac{x_p}{R}$$



$$E = e^{-i\omega t}\int_{-D/2}^{D/2}\frac{\mathcal{E}_L}{\sqrt{(x_p - x)^2 + y_p^2 + z_p^2}}e^{i\frac{2\pi}{\lambda}\sqrt{(x_p - x)^2 + y_p^2 + z_p^2}}dy$$

$$r = \sqrt{(x_p - x)^2 + y_p^2 + z_p^2}$$

$$= \sqrt{R^2 - x_p^2 + (x_p - x)^2} = \sqrt{R^2 + x^2 - 2x_p x}$$

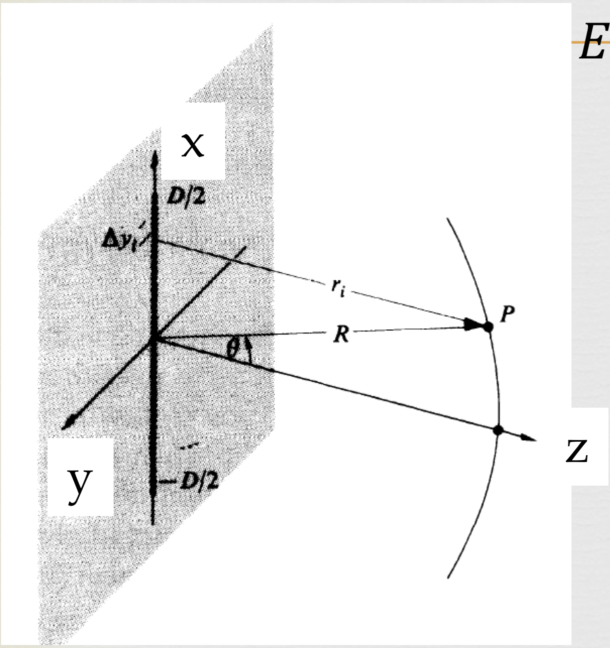
$$= R\sqrt{1 + \frac{x^2}{R^2} - \frac{2x_p x}{R^2}} \cong R\sqrt{1 - \frac{2x_p x}{R^2}} \cong R\left(1 - \frac{2x_p x}{2R^2}\right)$$

$$= R - \frac{x_p x}{R} = R - y\sin\theta$$

Fraunhore Diffraction

$$R = \sqrt{x_p^2 + y_p^2 + z_p^2} >> D \qquad \sin\theta = \frac{x_p}{R}$$

$$E = e^{-i\omega t}\int_{-D/2}^{D/2} \frac{\mathcal{E}_L}{\sqrt{(x_p - x)^2 + y_p^2 + z_p^2}} e^{ik\sqrt{(x_p-x)^2+y_p^2+z_p^2}}dx$$



$$r = \sqrt{(x_p - x)^2 + y_p^2 + z_p^2} \cong R - x\sin\theta$$

$$E = \frac{\mathcal{E}_L}{R} e^{-i\omega t}\int_{-D/2}^{D/2} e^{i(kR - kx\sin\theta)}dx$$

$$= \frac{\mathcal{E}_L}{R} e^{ikR} e^{-i\omega t}\int_{-D/2}^{D/2} e^{-ikx\sin\theta}dx$$

$$= \frac{\mathcal{E}_L}{R} e^{ikR} e^{-i\omega t}\left[\frac{e^{-ikx\sin\theta}}{-ik\sin\theta}\right]_{-D/2}^{D/2} = \frac{\mathcal{E}_L}{R} e^{ikR} e^{-i\omega t}\frac{e^{-i(\frac{kD}{2})\sin\theta} - e^{i(\frac{kD}{2})\sin\theta}}{-ik\sin\theta} = \frac{\mathcal{E}_L}{R} e^{ikR} e^{-i\omega t}\frac{-2i\sin\left[(\frac{kD}{2})\sin\theta\right]}{-ik\sin\theta}$$

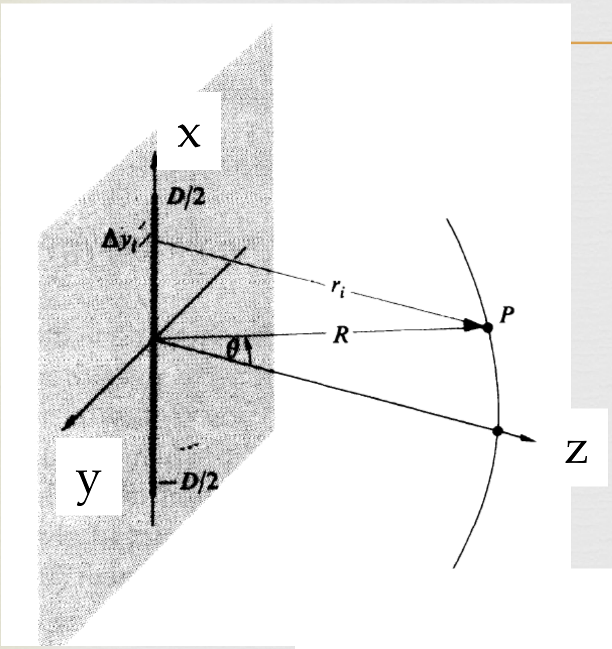
$$= \frac{D\mathcal{E}_L}{R} e^{ikR} e^{-i\omega t}\frac{\sin\left[(\frac{kD}{2})\sin\theta\right]}{\frac{kD}{2}\sin\theta} = \frac{D\mathcal{E}_L}{R} e^{ikR} e^{-i\omega t}\,\mathrm{sinc}\,\beta \qquad \beta = (\frac{kD}{2})\sin\theta$$

Fraunhore Diffraction

R = $\sqrt{x_p^2 + y_p^2 + z_p^2}$ >> D

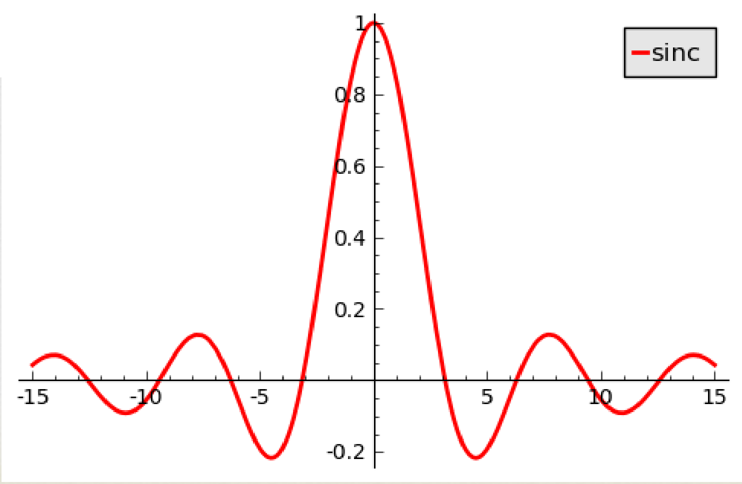
$$\sin\theta = \frac{x_p}{R}$$

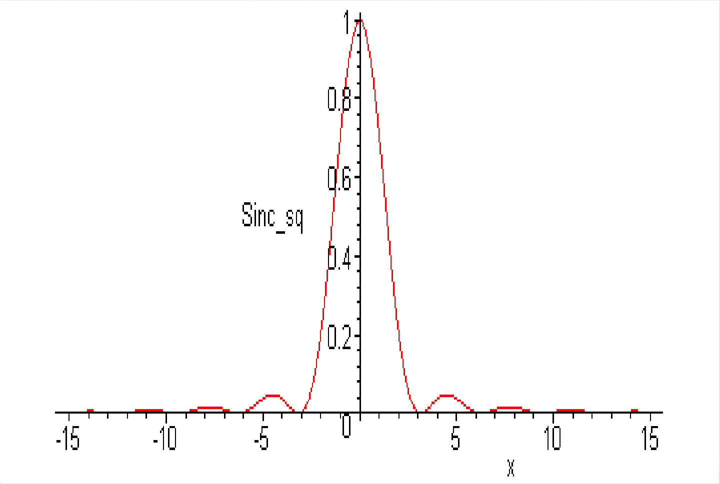
$$\beta = (\frac{kD}{2}) \sin\theta$$



$$E = \frac{D\mathcal{E}_L}{R} e^{ikR} e^{-i\omega t} \operatorname{sinc}\beta$$

$$I(\theta) = \frac{1}{2}E^*E = \frac{1}{2}\left(\frac{D\mathcal{E}_L}{R}\right)^2 \operatorname{sinc}^2\beta$$





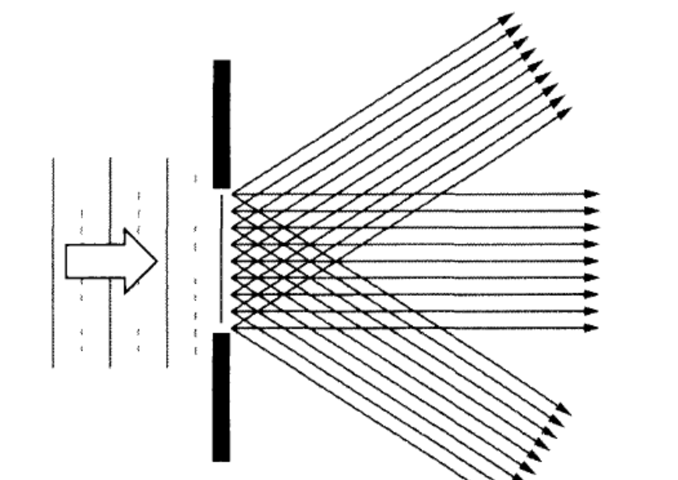
Single-Slit Fraunhor Diffraction

R >> D

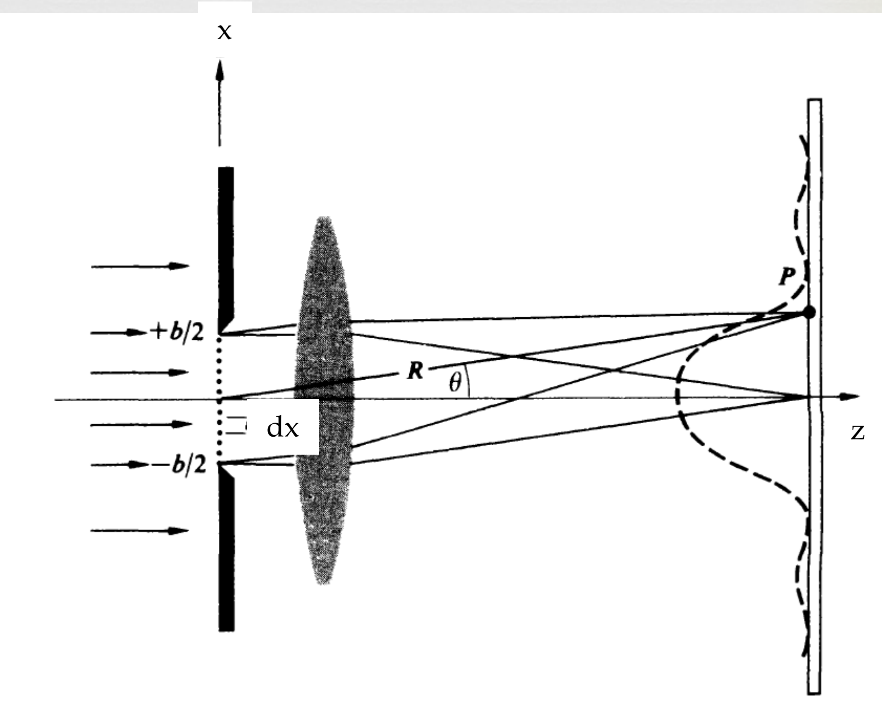
$$\sin\theta = \frac{y_p}{R}$$

$$\beta = (\frac{kD}{2})\sin\theta$$

$$I(\theta) = \frac{1}{2}E^*E = \frac{1}{2}\left(\frac{D\mathcal{E}_L}{R}\right)^2 \operatorname{sinc}^2\beta$$



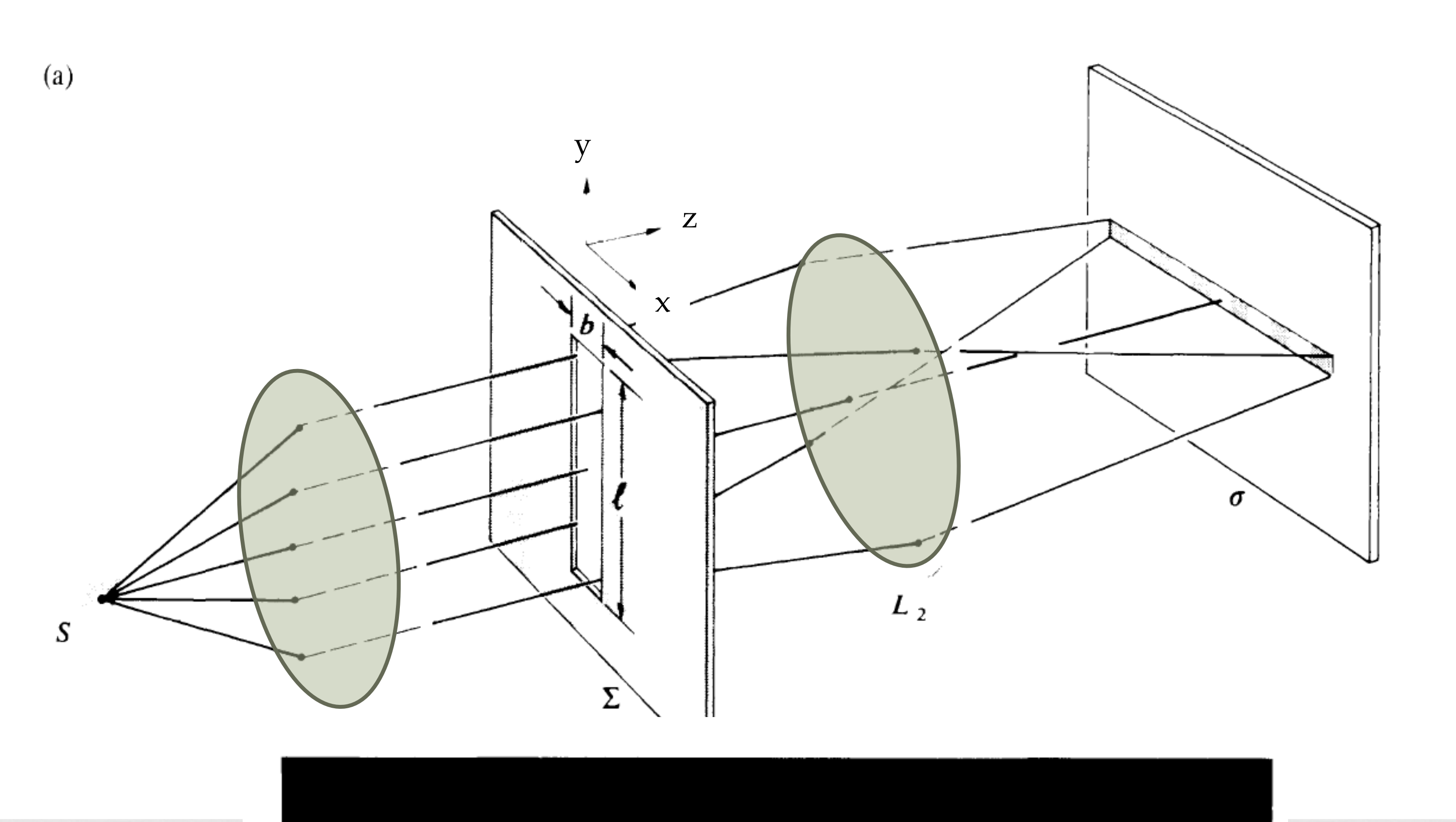
(c)

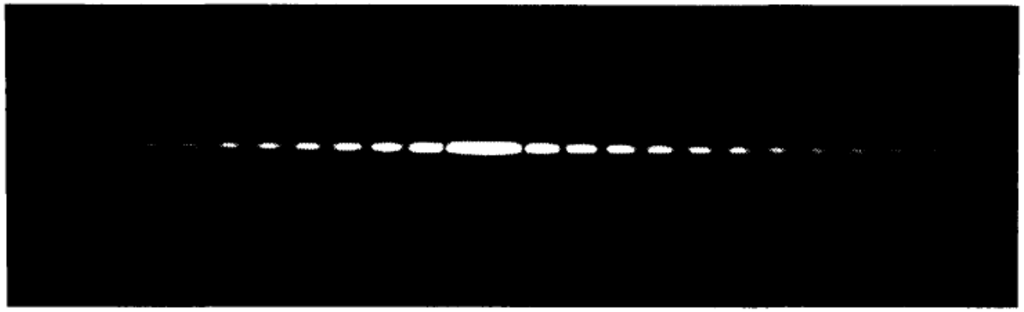


(e)

Single-Slit Fraunhor Diffraction

$$I(\theta) = \frac{1}{2}E^*E = \frac{1}{2}\left(\frac{D\mathcal{E}_L}{R}\right)^2 \operatorname{sinc}^2 \beta$$





Single-Slit Fraunhor Diffraction

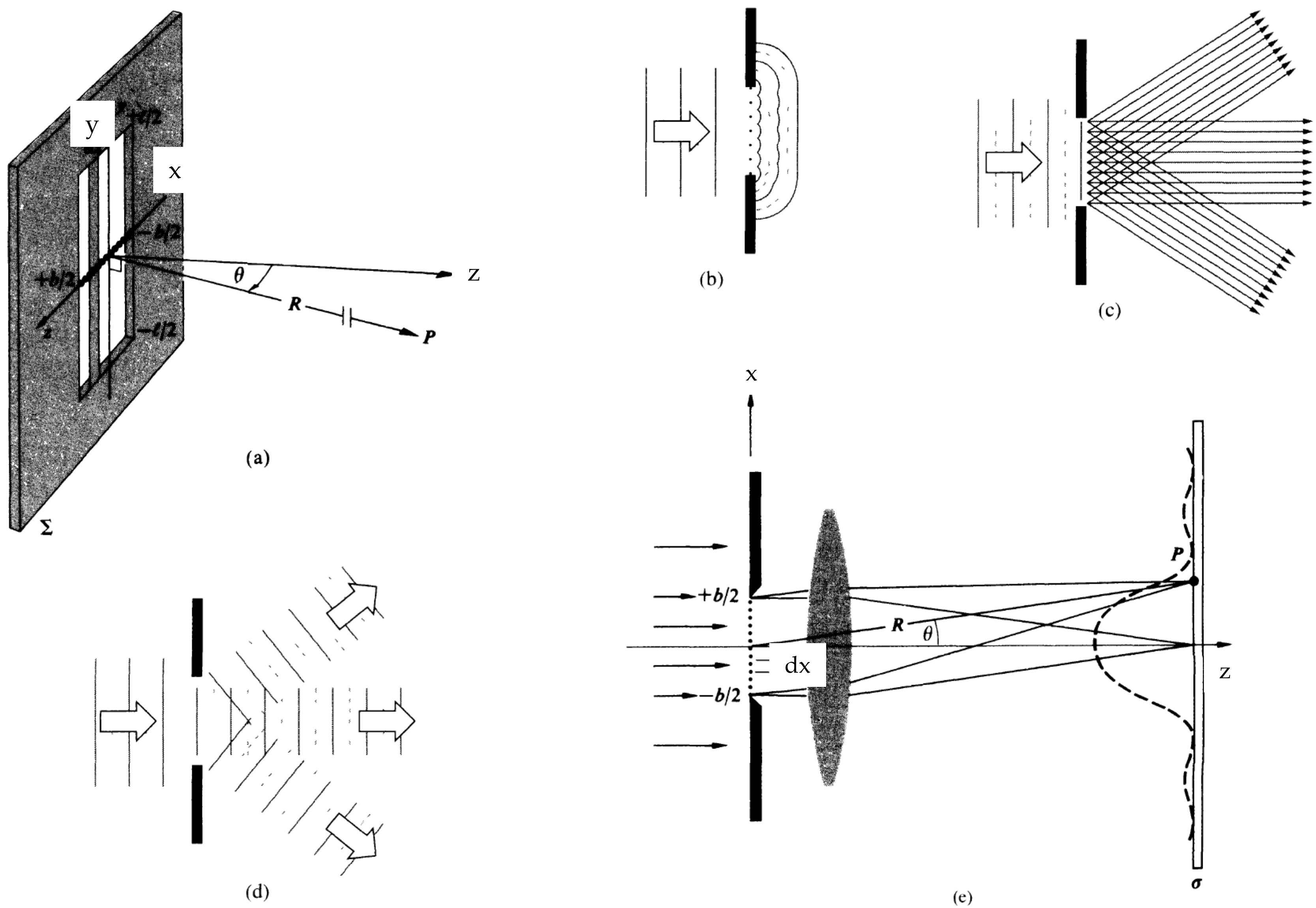


Figure 10.7 (a) Point P on σ is essentially infinitely far from Σ. (b) Huygens wavelets emitted across the aperture.

Single-Slit Diffraction

sidiary maxima will be observable. The extrema of $I(\theta)$ occur at values of β that cause $dI/d\beta$ to be zero, that is,

$$\frac{dI}{d\beta} = I(0)\frac{2\sin\beta(\beta\cos\beta - \sin\beta)}{\beta^3} = 0 \quad (10.19)$$

The irradiance has minima, equal to zero, when $\sin\beta = 0$, whereupon

$$\beta = \pm\pi,\ \pm 2\pi,\ \pm 3\pi, \ldots \quad (10.20)$$

It also follows from Eq. (10.19) that when

$$\beta\cos\beta - \sin\beta = 0$$

$$\tan\beta = \beta \quad (10.21)$$

so that $I(\theta)$ must have subsidiary maxima at these values of β (viz, $\pm 1.4303\pi$, $\pm 2.4590\pi$, $\pm 3.4707\pi, \ldots$).

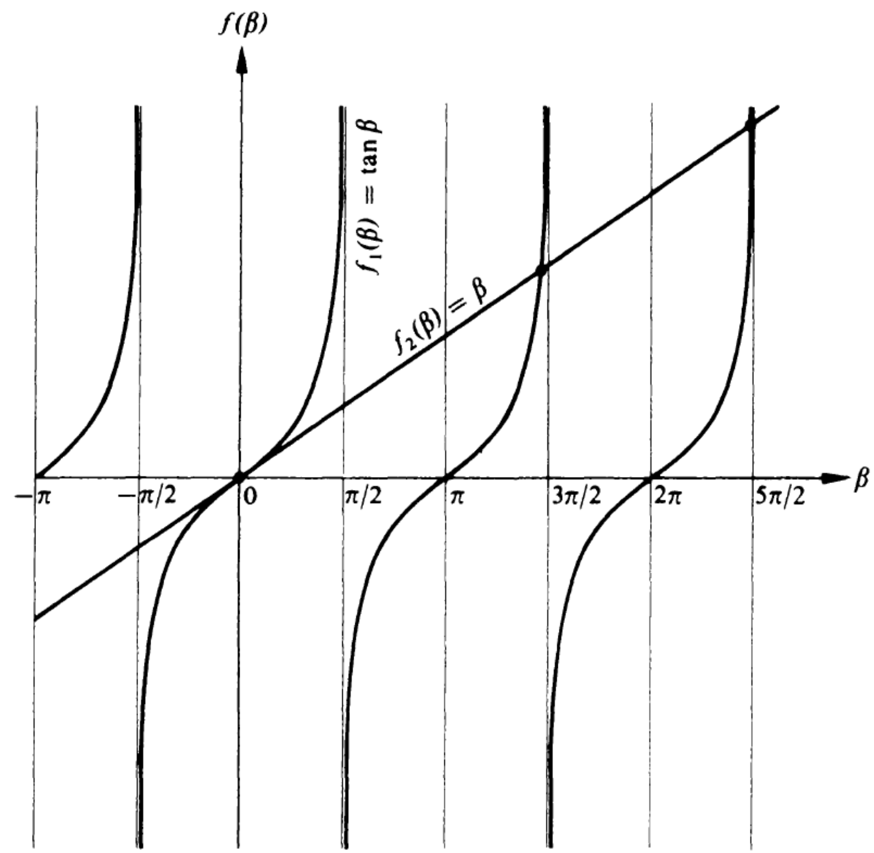
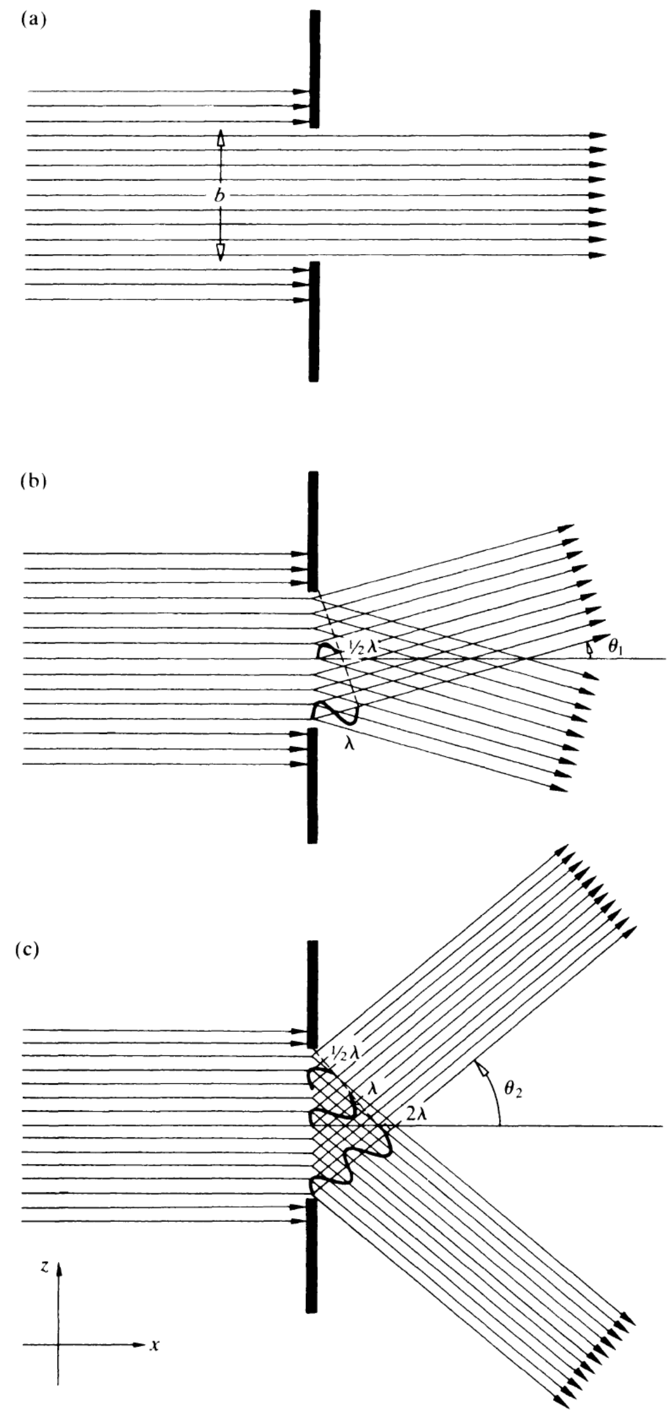


Figure 10.8 The points of intersection of the two curves are the solutions of Eq. (10.21).


(a)
b
(b)
½λ
θ1
λ
(c)
½λ
λ
2λ
θ2
z
x

$$\beta_m = \left(\frac{kD}{2}\right)\sin\theta_m = \frac{\pi D}{\lambda}\sin\theta_m = m\pi \qquad D\sin\theta_m = m\,\lambda$$

$$m = \pm 1, \pm 2, \pm 3, \ldots$$

Zeros of irradiance

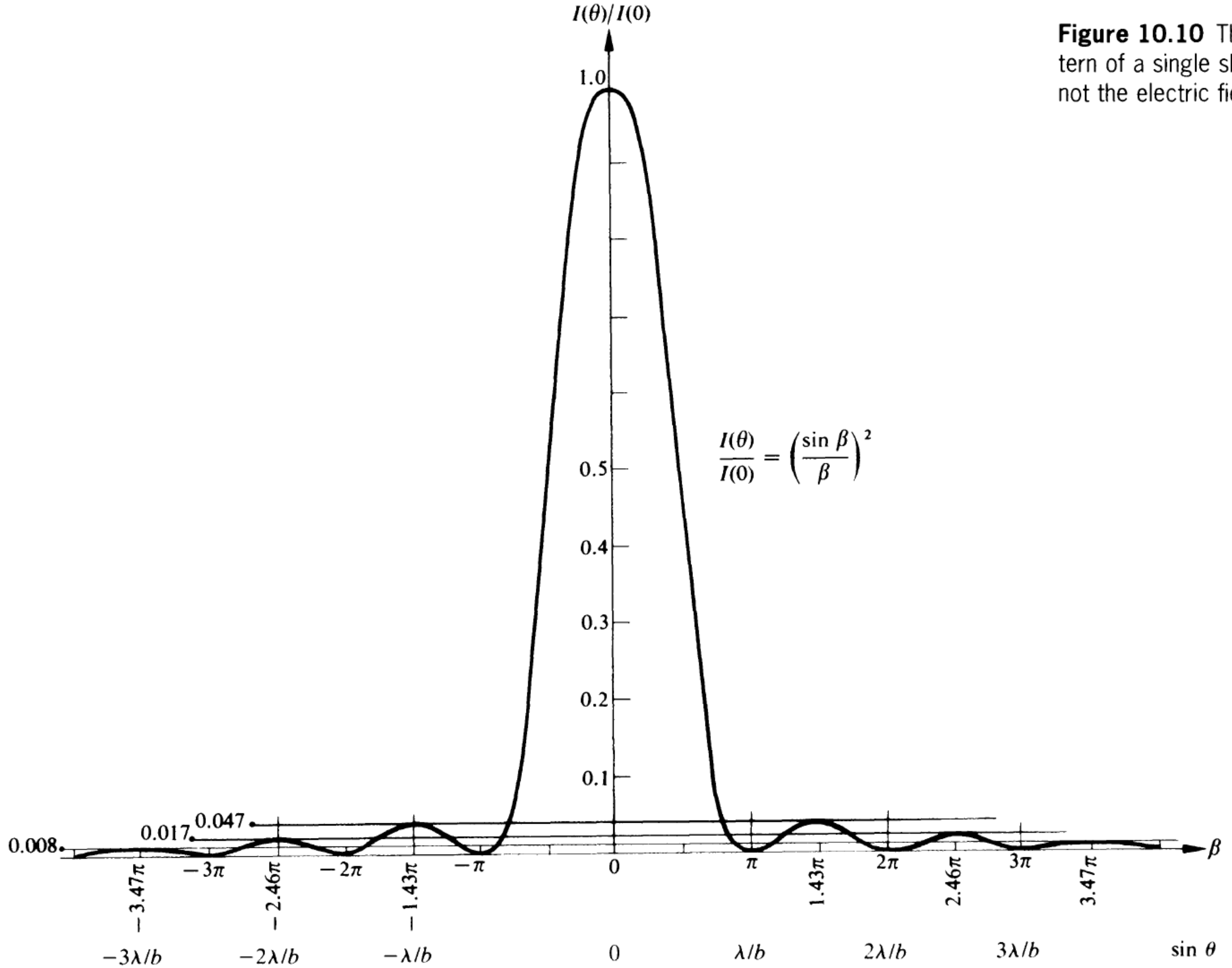
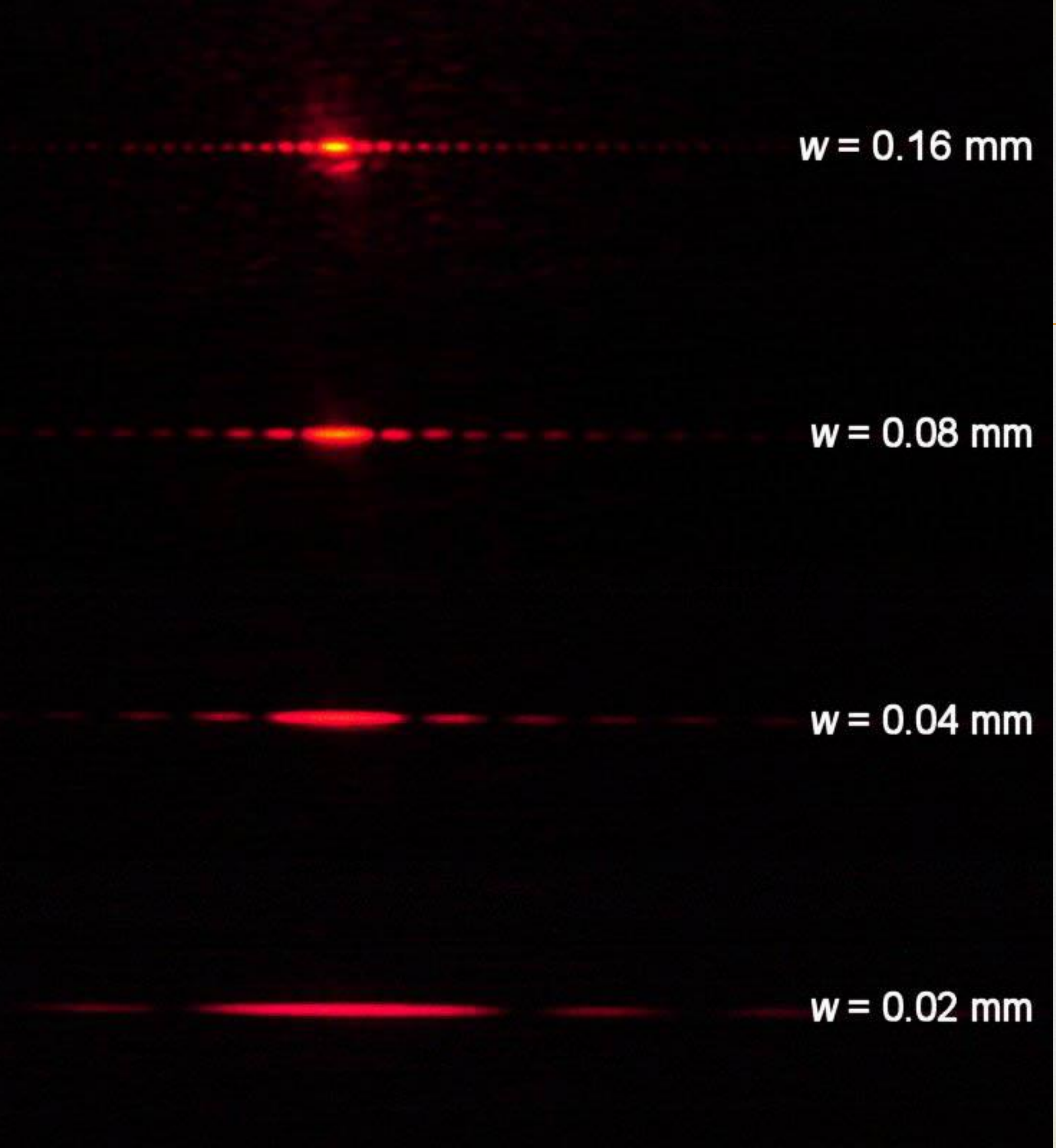


Figure 10.10 The Fraunhofer diffraction pattern of a single slit. This is the irradiance (and not the electric field) distribution

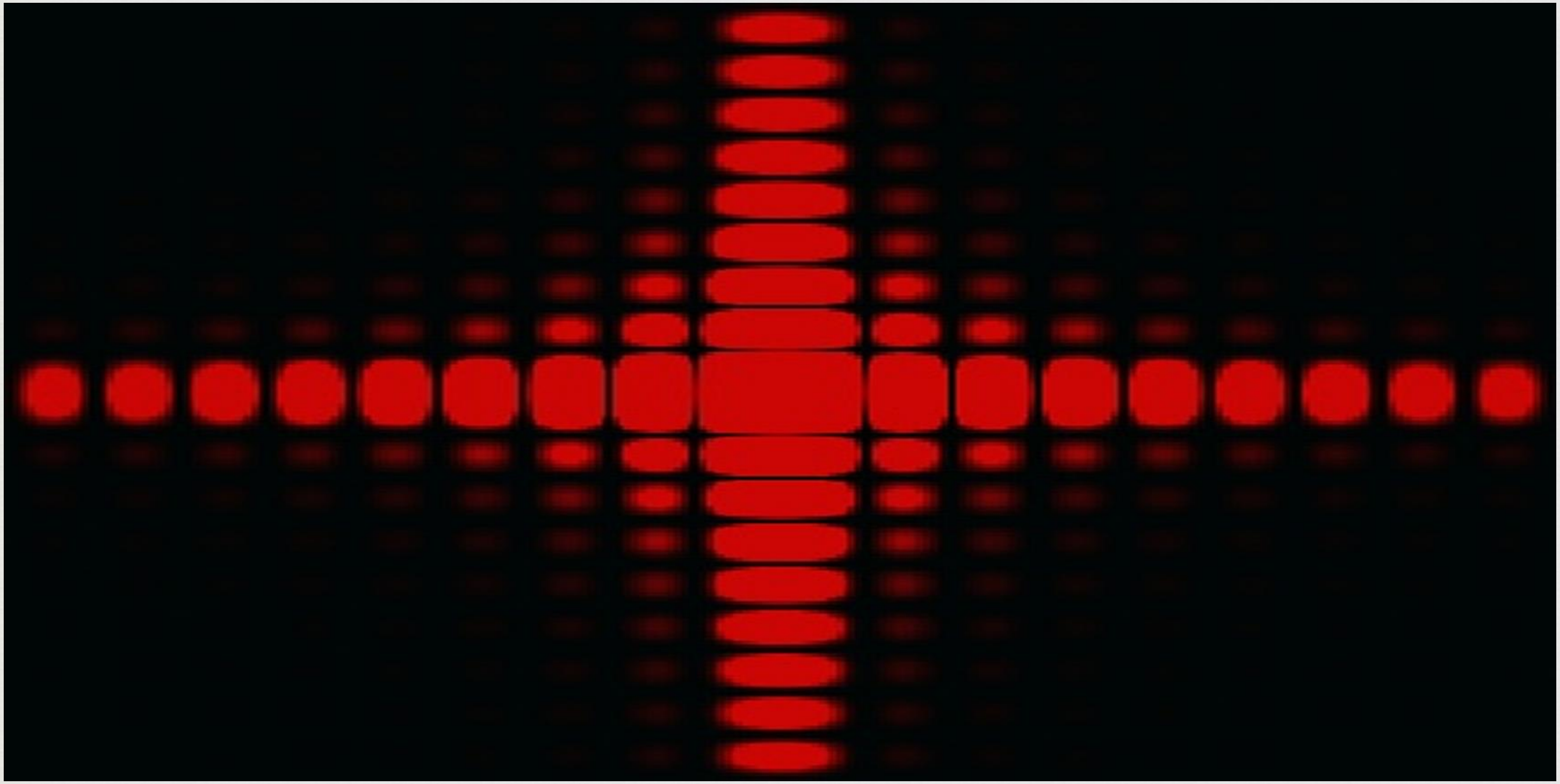


$$\operatorname{sinc}^2 \beta$$

$$\beta = \left(\frac{kD}{2}\right) \sin\theta$$

$$D \sin\theta_m = m\,\lambda$$

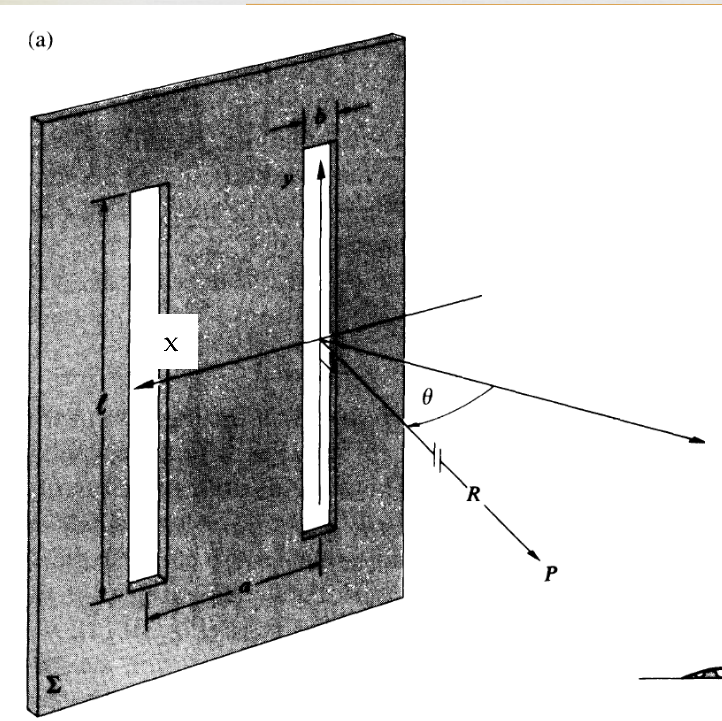
Two-Dimensinal Slit



$I = I_0 \operatorname{sinc}^2 \beta_x \operatorname{sinc}^2 \beta_y$

$\beta_{x,y} = (\frac{kD_{x,y}}{2}) \sin\theta_{x,y}$

The Double Slit



Recall: single slit $E_1 = \frac{D\mathcal{E}_L}{R} e^{ikR} e^{-i\omega t} \operatorname{sinc}\beta$

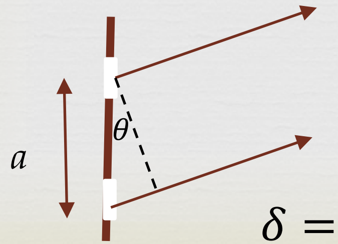
Diffraction + Interference

$$E = E_1 + E_2 \;= E_1 + E_1 e^{i\delta} = E_1 e^{\frac{i\delta}{2}} \left(e^{-\frac{i\delta}{2}} + e^{-\frac{i\delta}{2}}\right)$$

$$= 2E_1\, e^{\frac{i\delta}{2}} \cos\frac{\delta}{2}$$

$$I = \frac{1}{2} E^* E \;= 2E_1^* E_1 cos^2\frac{\delta}{2} = 4 I_1 cos^2\frac{\delta}{2}$$

$$= 4 I_0 \frac{sin^2\beta}{\beta^2} cos^2\frac{\delta}{2}$$



$$\delta = ka \sin\theta$$

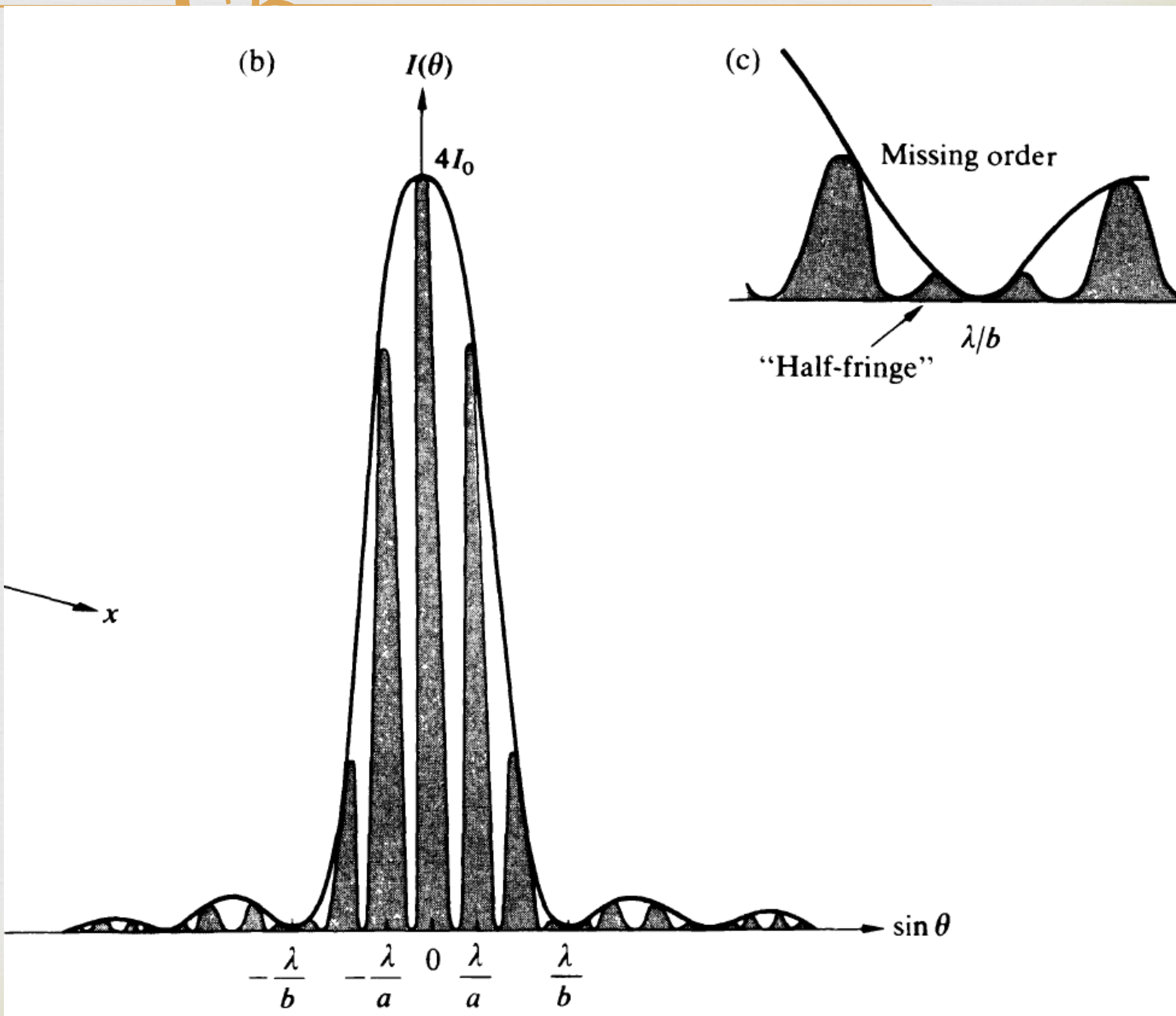
The Double Slit

$$I = 4I_0 \frac{sin^2 \beta}{\beta^2} cos^2 \frac{\delta}{2}$$

$$= 4I_0 \frac{sin^2 \beta}{\beta^2} cos^2 \alpha$$

$$\beta = \frac{kD}{2} \sin\theta$$

$$\alpha = \delta/2 = \frac{ka}{2} \sin\theta$$



The Single-Slit and Double-Slit