PHYS 3038 Optics
L16 Interference
Reading Material: Ch9.7-8

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2015, the Year of Light

9.7 Applications

- Coating
  - Anti-reflection (AR) coating
  - Dielectric mirror
  - Dichroic mirror

Example

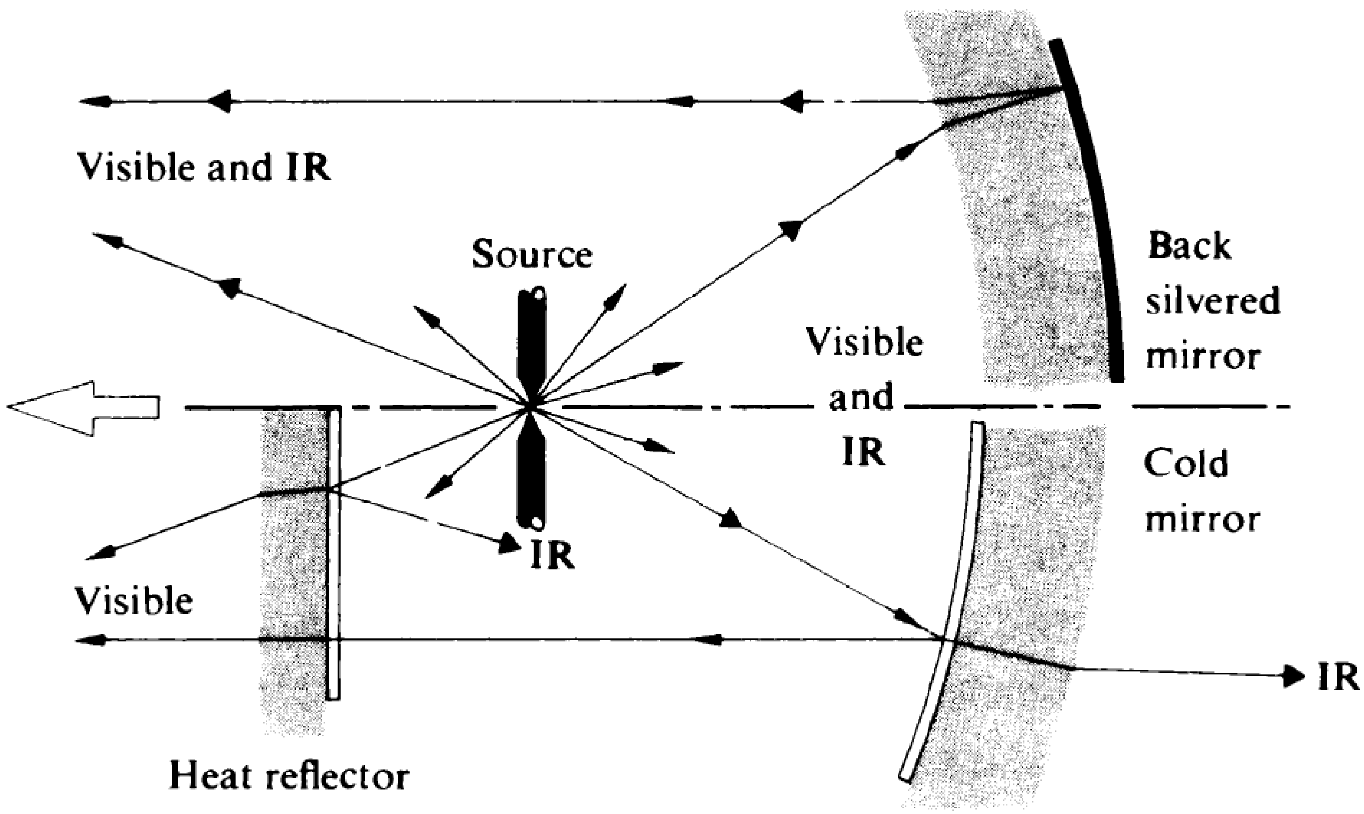
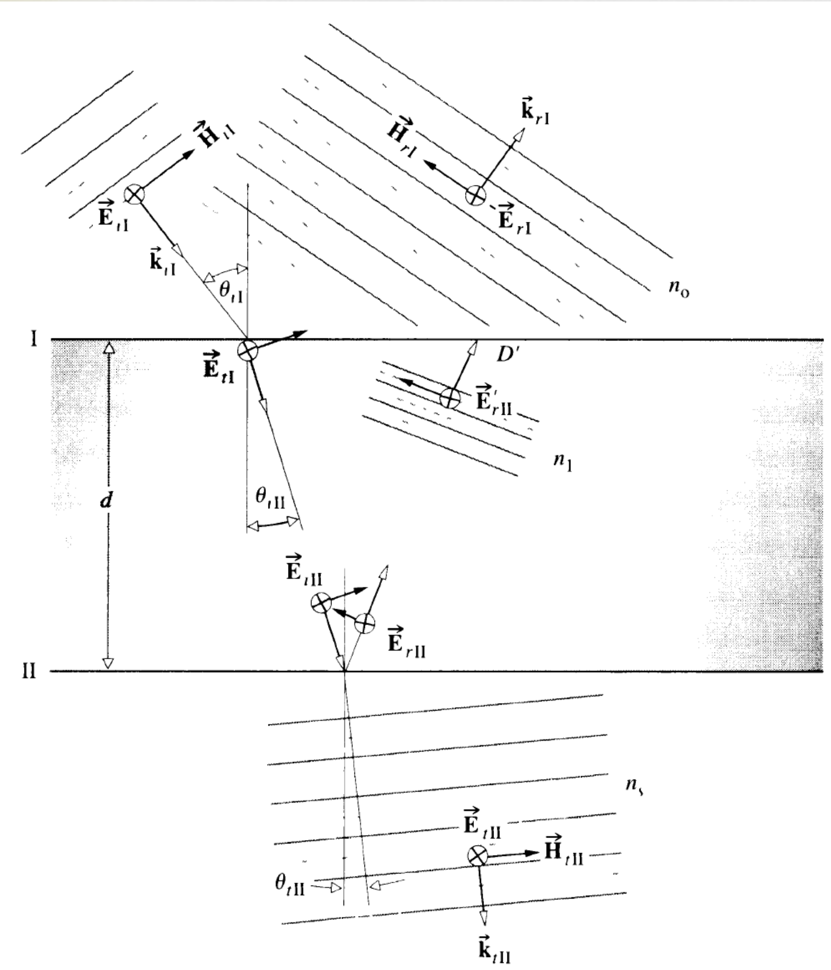


Figure 9.48 A composite drawing showing an ordinary system in the top half and a coated one in the bottom.

Mathematical Treatment



$$\begin{bmatrix} E_I \\ H_I \end{bmatrix} = \begin{bmatrix} \cos k_0 h & (i \sin k_0 h)/Y_1 \\ Y_1 i \sin k_0 h & \cos k_0 h \end{bmatrix} \begin{bmatrix} E_{II} \\ H_{II} \end{bmatrix} \quad (9.91)$$

$$\begin{bmatrix} E_I \\ H_I \end{bmatrix} = \mathcal{M}_I \begin{bmatrix} E_{II} \\ H_{II} \end{bmatrix} \quad (9.92)$$

When E is perpendicular to the plane-of-incidence

$$Y_1 \equiv \sqrt{\frac{\epsilon_0}{\mu_0}}\, n_1 \cos \theta_{iII}$$

When E is in the plane-of-incidence

$$Y_1 \equiv \sqrt{\frac{\epsilon_0}{\mu_0}}\, n_1 / \cos \theta_{iII}$$

For multiple layers:

$$\begin{bmatrix} E_I \\ H_I \end{bmatrix} = \mathcal{M}_I \mathcal{M}_{II} \cdots \mathcal{M}_p \begin{bmatrix} E_{(p+1)} \\ H_{(p+1)} \end{bmatrix}$$

Single Layer

$$\begin{bmatrix} E_{\mathrm{I}} \\ H_{\mathrm{I}} \end{bmatrix} = \mathcal{M}_{\mathrm{I}} \begin{bmatrix} E_{\mathrm{II}} \\ H_{\mathrm{II}} \end{bmatrix}$$

$$E_I = E_{iI} + E_{rI} = E_{iI} + rE_{iI}$$
$$= (1+r)E_{iI}$$

$$H_I = \sqrt{\frac{\epsilon_0}{\mu_0}}(E_{iI} - E_{rI})n_0 \cos\theta_{iI}$$
$$= \sqrt{\frac{\epsilon_0}{\mu_0}}(1-r)n_0 E_{iI}\cos\theta_{iI}$$

$$E_{II} = E_{tII} = tE_{iI}$$

$$H_{II} = \sqrt{\frac{\epsilon_0}{\mu_0}} E_{tII} n_s \cos\theta_{iII}$$
$$= \sqrt{\frac{\epsilon_0}{\mu_0}} tE_{iI} n_s \cos\theta_{iI}$$

setting

$$\mathrm{Y}_0 = \sqrt{\frac{\epsilon_0}{\mu_0}}\, n_0 \cos\theta_{i\mathrm{I}}$$

and

$$\mathrm{Y}_s = \sqrt{\frac{\epsilon_0}{\mu_0}}\, n_s \cos\theta_{t\mathrm{II}}$$

we obtain

$$\begin{bmatrix} (E_{i\mathrm{I}} + E_{r\mathrm{I}}) \\ (E_{i\mathrm{I}} - E_{r\mathrm{I}})\mathrm{Y}_0 \end{bmatrix} = \mathcal{M}_1 \begin{bmatrix} E_{t\mathrm{II}} \\ E_{t\mathrm{II}}\mathrm{Y}_s \end{bmatrix}$$

When the matrices are expanded, the last relation becomes

$$1 + r = m_{11}t + m_{12}\mathrm{Y}_s t$$

and

$$(1 - r)\mathrm{Y}_0 = m_{21}t + m_{22}\mathrm{Y}_s t$$

inasmuch as

$$r = E_{r\mathrm{I}}/E_{i\mathrm{I}} \quad \text{and} \quad t = E_{t\mathrm{II}}/E_{i\mathrm{I}}$$

Consequently,

$$r = \frac{\mathrm{Y}_0 m_{11} + \mathrm{Y}_0\mathrm{Y}_s m_{12} - m_{21} - \mathrm{Y}_s m_{22}}{\mathrm{Y}_0 m_{11} + \mathrm{Y}_0\mathrm{Y}_s m_{12} + m_{21} + \mathrm{Y}_s m_{22}} \tag{9.97}$$

and

$$t = \frac{2\mathrm{Y}_0}{\mathrm{Y}_0 m_{11} + \mathrm{Y}_0\mathrm{Y}_s m_{12} + m_{21} + \mathrm{Y}_s m_{22}} \tag{9.98}$$

Antireflection Coating Single Layer

$\theta_i = 0$

$$r_1 = \frac{n_1(n_0 - n_s)\cos k_0 h + i(n_0 n_s - n_1^2)\sin k_0 h}{n_1(n_0 + n_s)\cos k_0 h + i(n_0 n_s + n_1^2)\sin k_0 h}$$

$$R_1 = \frac{n_1^2(n_0 - n_s)^2\cos^2 k_0 h + (n_0 n_s - n_1^2)^2\sin^2 k_0 h}{n_1^2(n_0 + n_s)^2\cos^2 k_0 h + (n_0 n_s + n_1^2)^2\sin^2 k_0 h}$$

$d = \lambda_f/4 \Longleftrightarrow k_0 h = \pi/2$

$$R_1 = \frac{(n_0 n_s - n_1^2)^2}{(n_0 n_s + n_1^2)^2} \tag{9.101}$$

which, quite remarkably, will equal zero when

$$n_1^2 = n_0 n_s \tag{9.102}$$

Antireflection Coating

Double Layer

For a double-layer, quarter-wavelength antireflection coating,

$$\mathcal{M} = \mathcal{M}_{\mathrm{I}}\mathcal{M}_{\mathrm{II}}$$

or more specifically

$$\mathcal{M} = \begin{bmatrix} 0 & i/Y_1 \\ iY_1 & 0 \end{bmatrix}\begin{bmatrix} 0 & i/Y_2 \\ iY_2 & 0 \end{bmatrix} \tag{9.103}$$

At normal incidence this becomes

$$\mathcal{M} = \begin{bmatrix} -n_2/n_1 & 0 \\ 0 & -n_1/n_2 \end{bmatrix} \tag{9.104}$$

Substituting the appropriate matrix elements into Eq. (9.97) yields r_2, which, when squared, leads to the reflectance

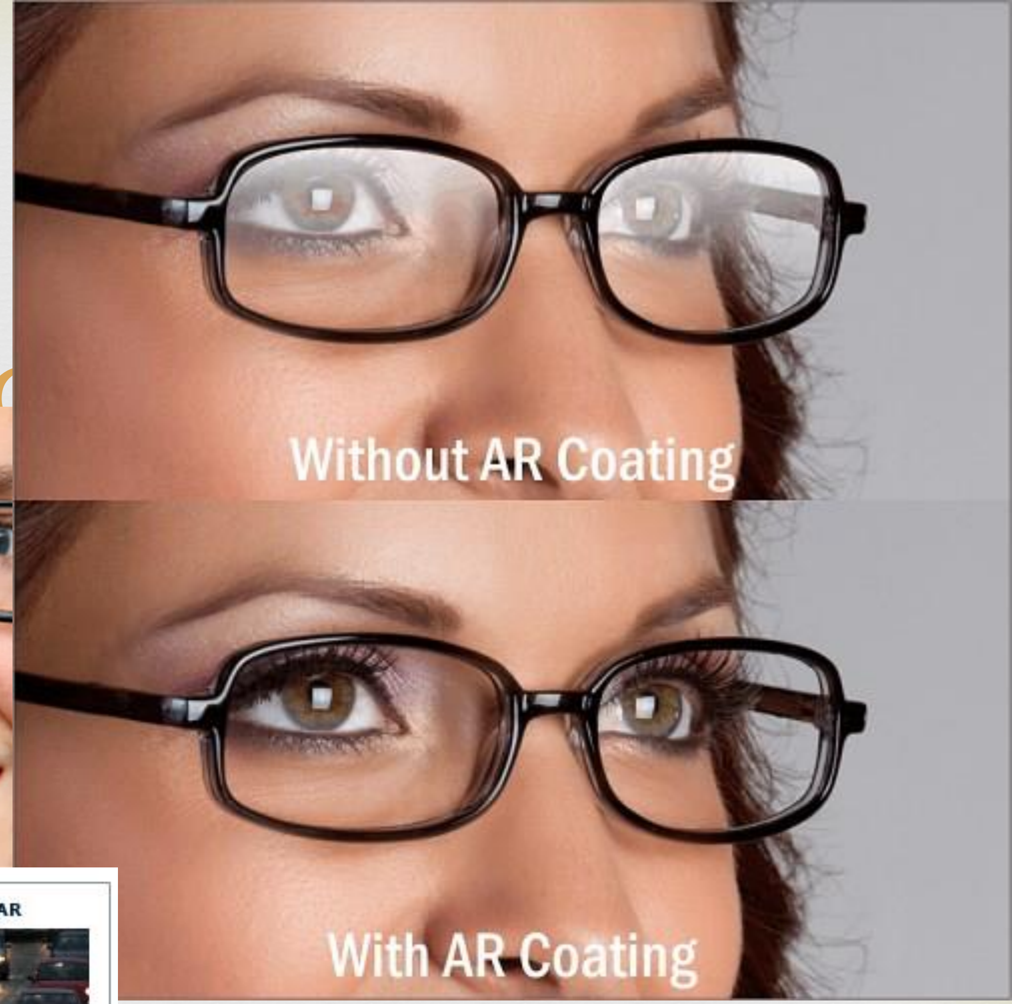
$$R_2 = \left[\frac{n_2^2 n_0 - n_s n_1^2}{n_2^2 n_0 + n_s n_1^2}\right]^2 \tag{9.105}$$

For R_2 to be exactly zero at a particular wavelength, we need

$$\left(\frac{n_2}{n_1}\right)^2 = \frac{n_s}{n_0} \tag{9.106}$$

This kind of film is referred to as a *double-quarter*, *single-minimum* coating. When n_1 and n_2 are as small as possible, the reflectance will have its single broadest minimum equal to zero at the chosen frequency. It should be clear from Eq. (9.106) that $n_2 > n_1$; accordingly, it is now common practice to designate a (glass)–(high index)–(low index)–(air) system as *gHLa*. Zirconium dioxide ($n = 2.1$), titanium dioxide ($n = 2.40$), and zinc sulfide ($n = 2.32$) are commonly used for *H*-layers, and magnesium fluoride ($n = 1.38$) and cerium fluoride ($n = 1.63$) often serve as *L*-layers.

AR Coating


Without AR Coating
With AR Coating


WITHOUT AR
WITH AR
WITHOUT AR
WITH AR

Multilayer Periodic Systems

The simplest kind of periodic system is the *quarter-wave stack*, which is made up of a number of quarter-wave layers. The periodic structure of alternately high- and low-index materials, illustrated in Fig. 9.50, is designated by

$$g(HL)^3a$$

Figure 9.51 illustrates the general form of a portion of the spectral reflectance for a few multilayer filters. The width of

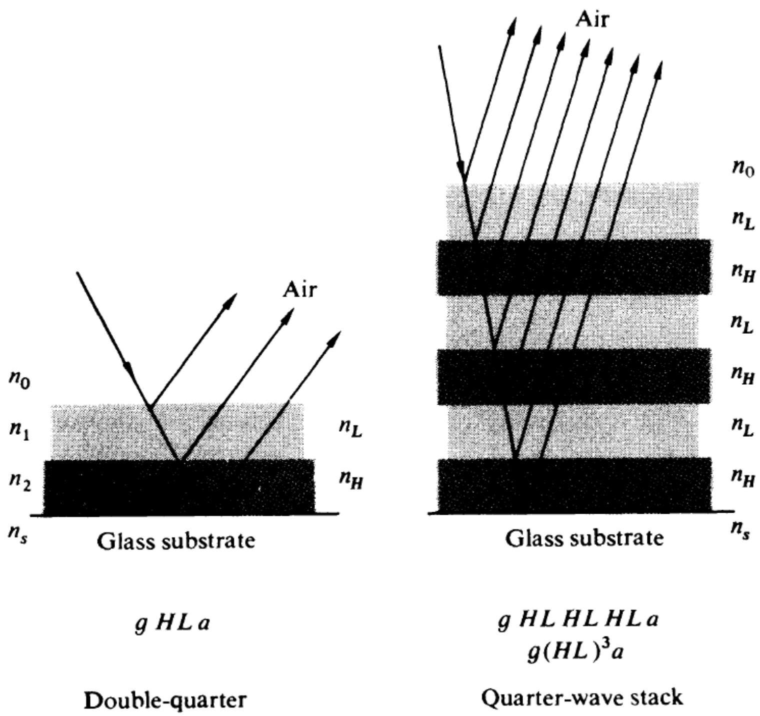


Figure 9.50 A periodic structure. Refraction has been omitted for simplicity.

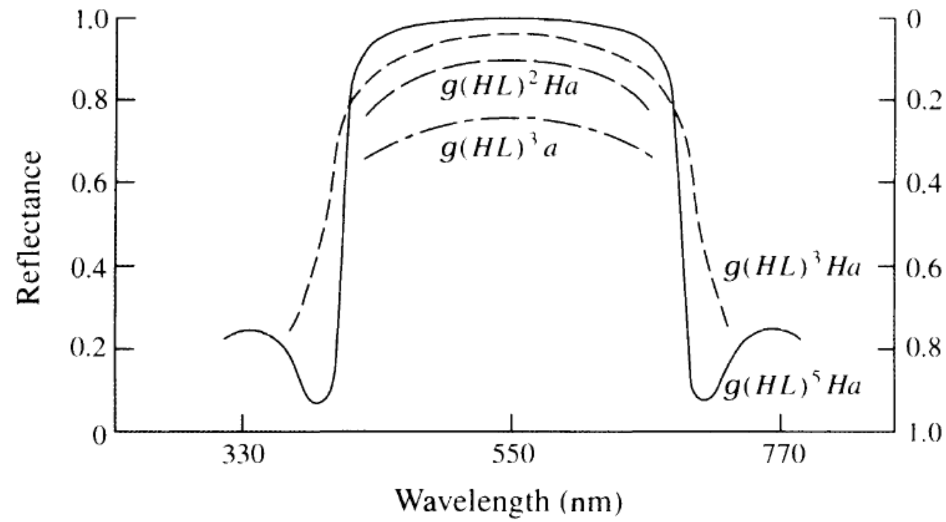
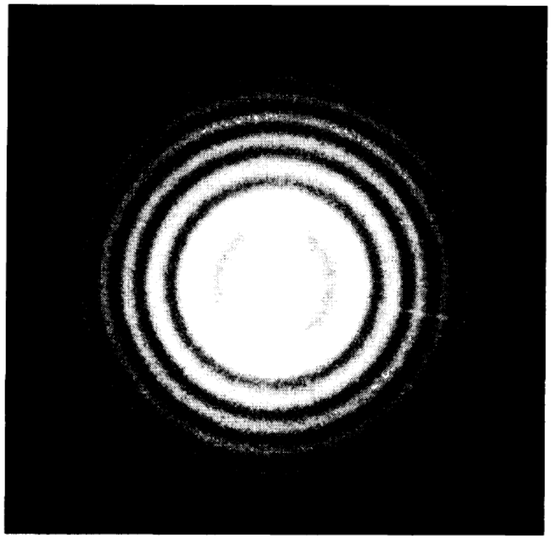


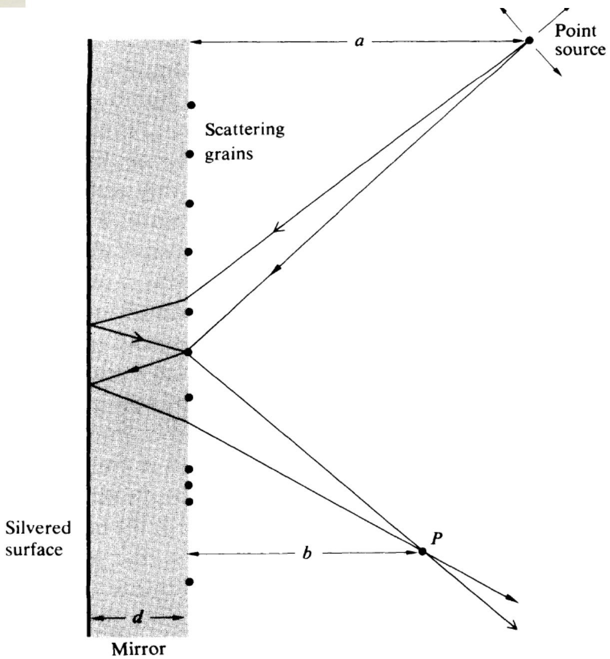
Figure 9.51 Reflectance and transmittance for several periodic structures.

9.8 Applications of Interferometry

- Scattered-Light Interference
- Twyman-Green Interferometer
- Rotating Sagnac Interferometer
- Radar Interferometry
- …
- …
- …

Scattered-Light Interference





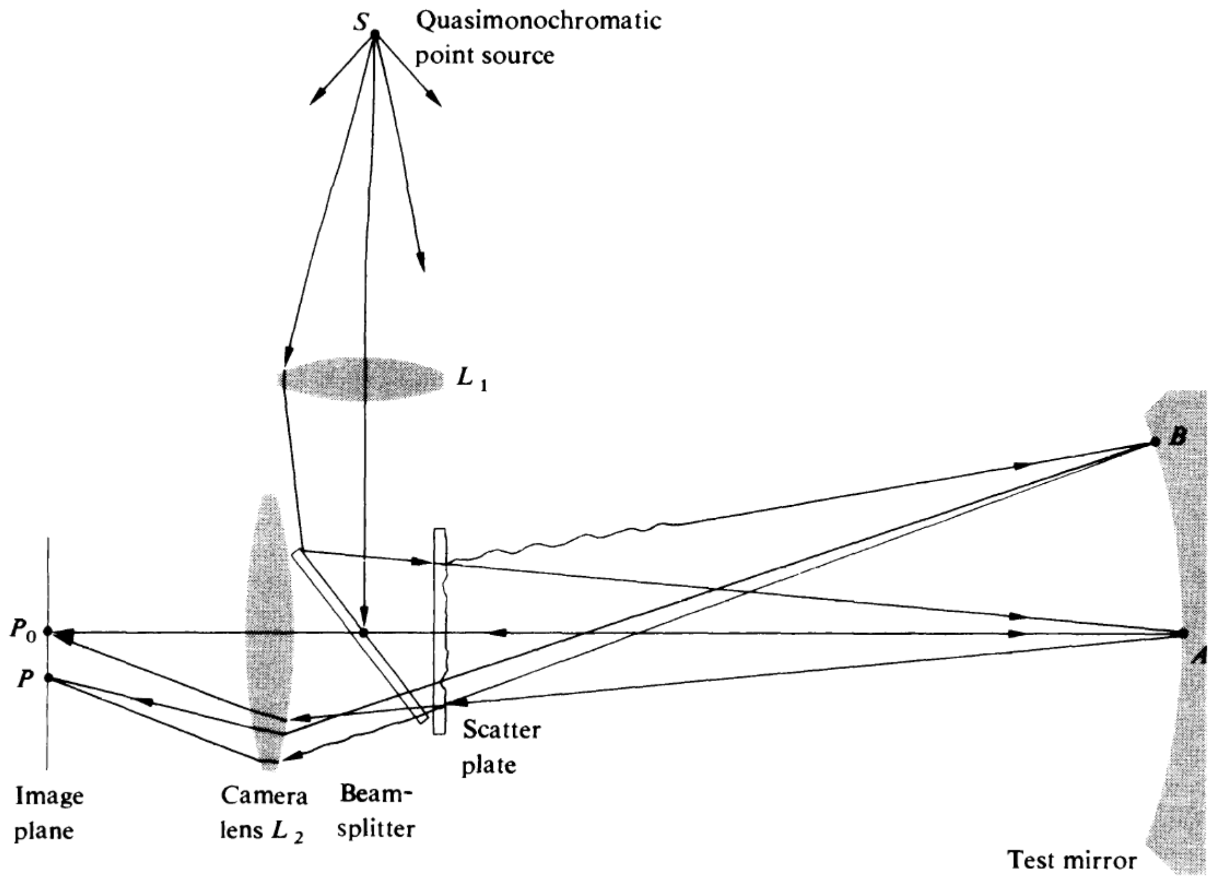
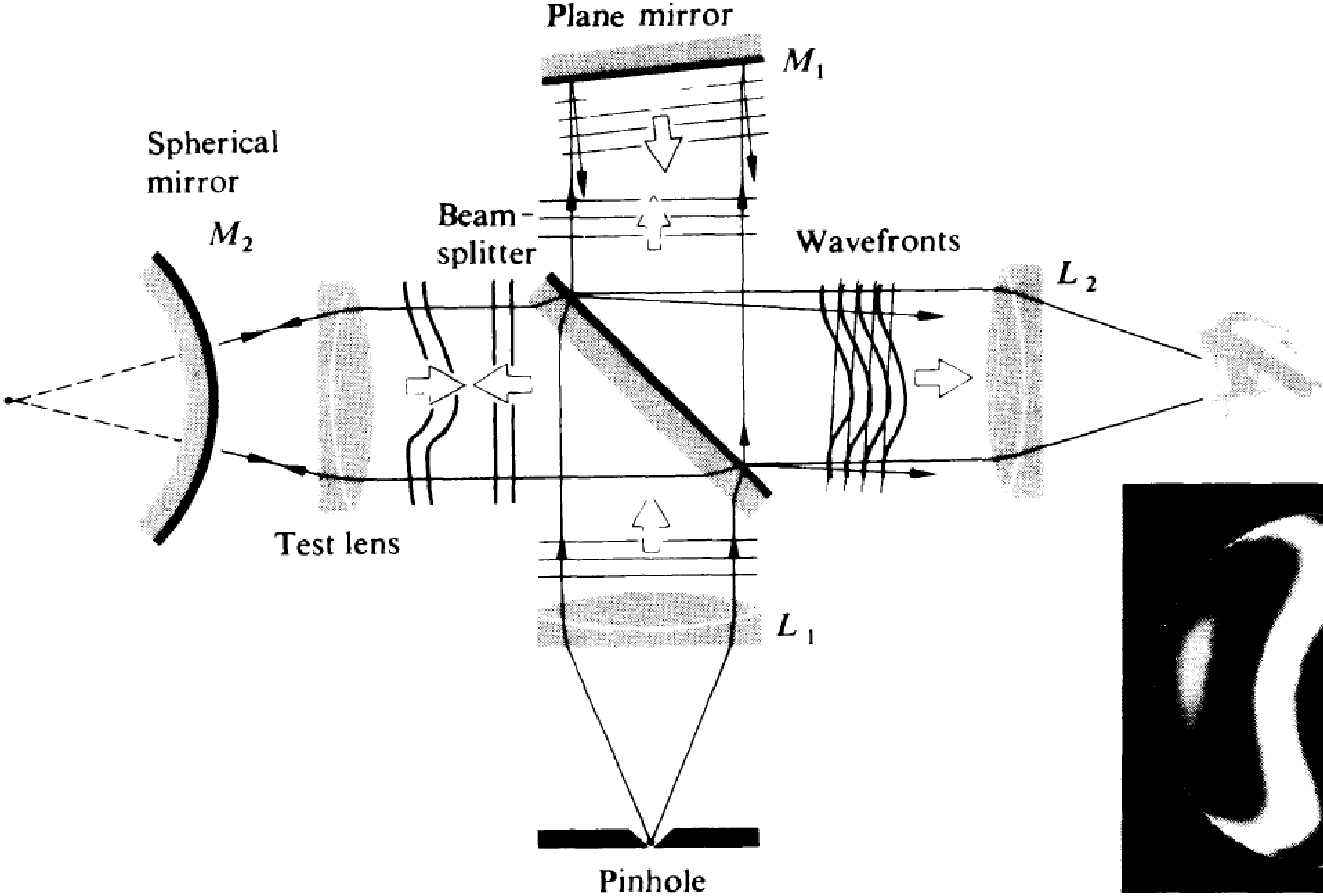
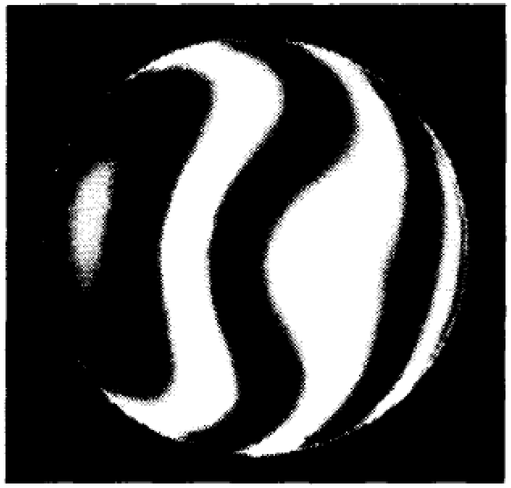


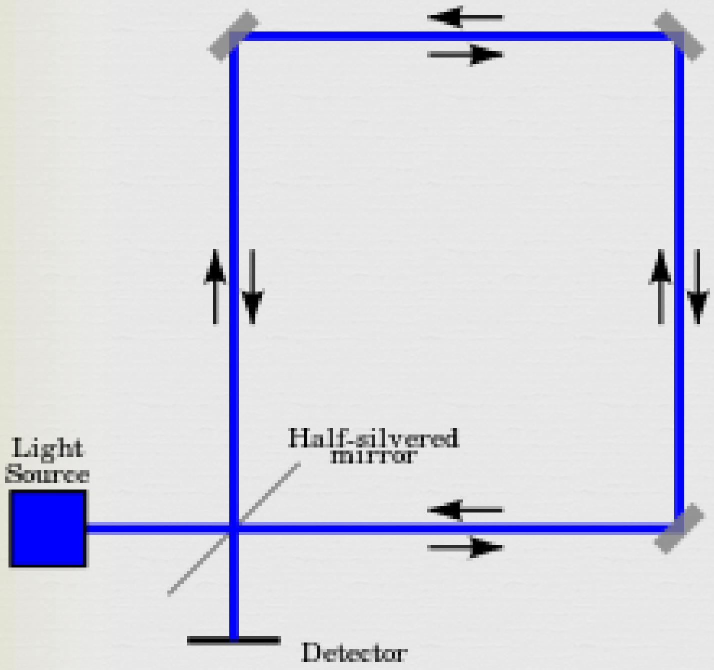
Figure 9.52 Interference of scattered light.

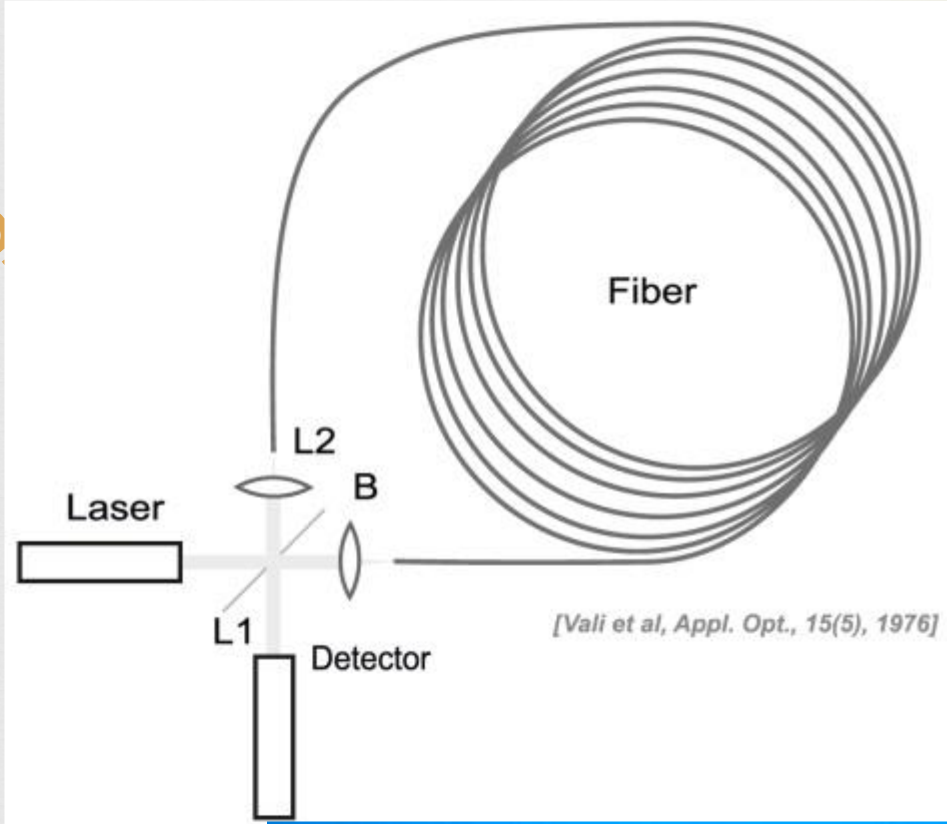
Twyman-Green Interferometer

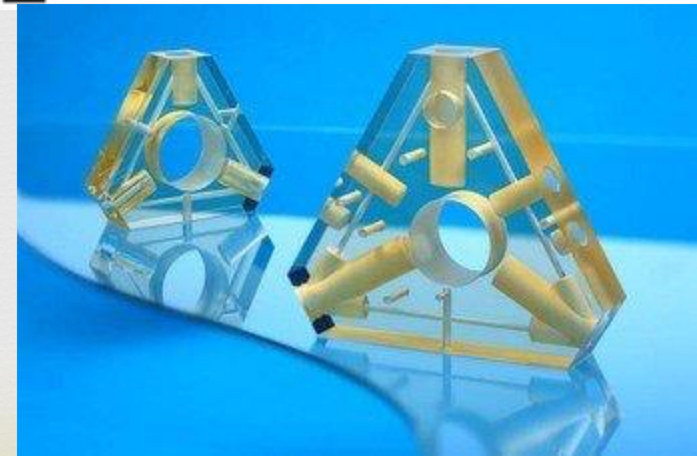




Rotating Sagnac Interferometer







Radar Interferometry

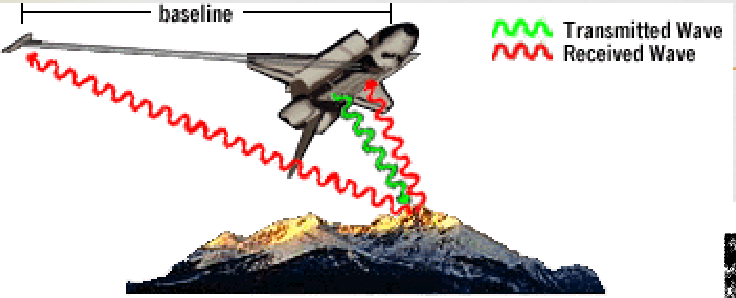
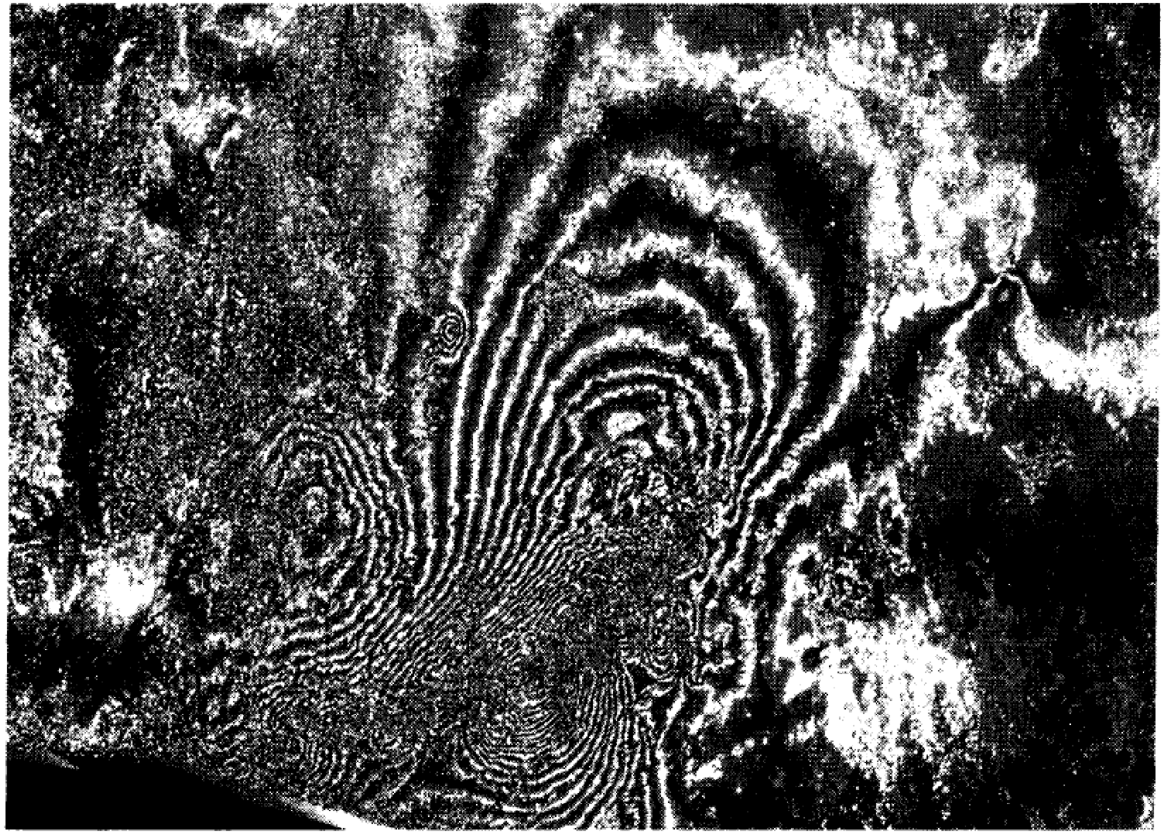


Figure 3: Two Receivers



https://www.youtube.com/watch?v=4rIHdcwlvlk

SAR Interferometer