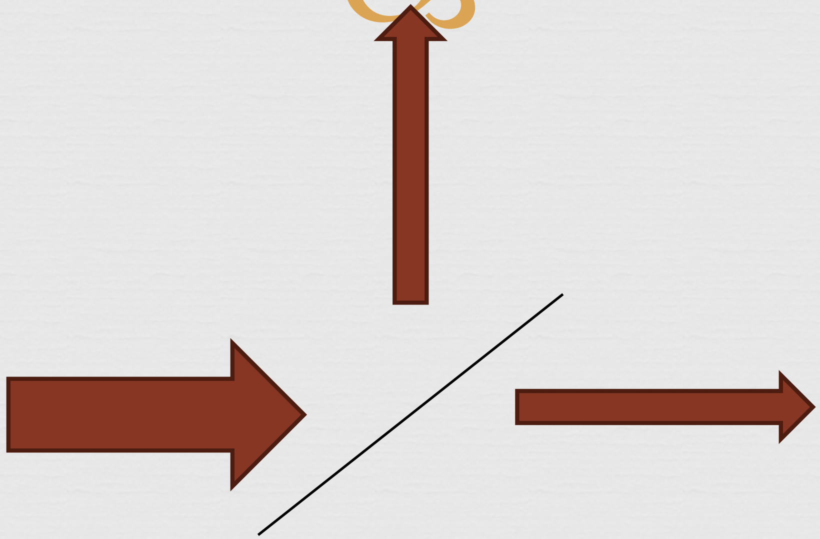
PHYS 3038 Optics
L14 Interference
Reading Material: Ch9.3-4

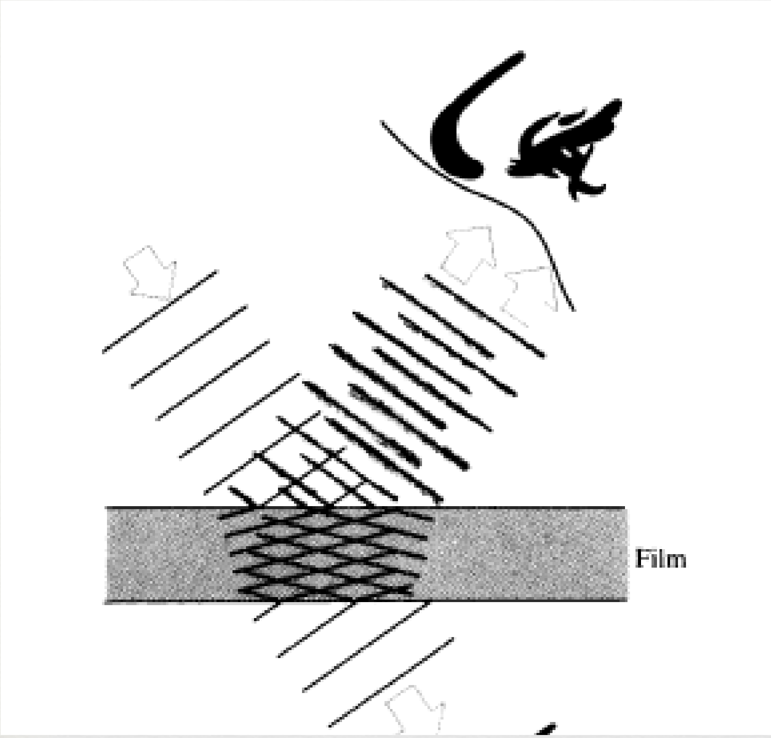
Shengwang Du

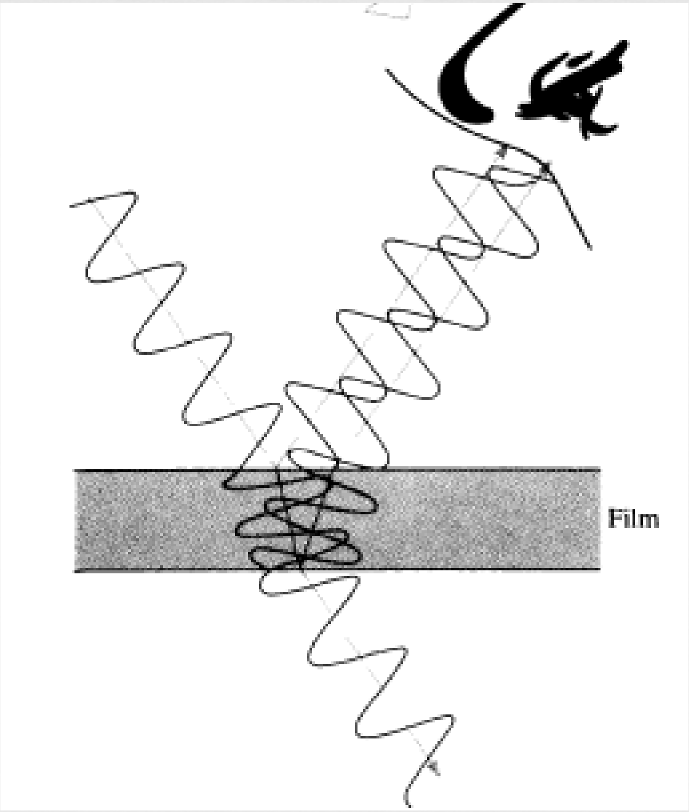
2015, the Year of Light

9.4 Amplitude-Splitting Interferometers

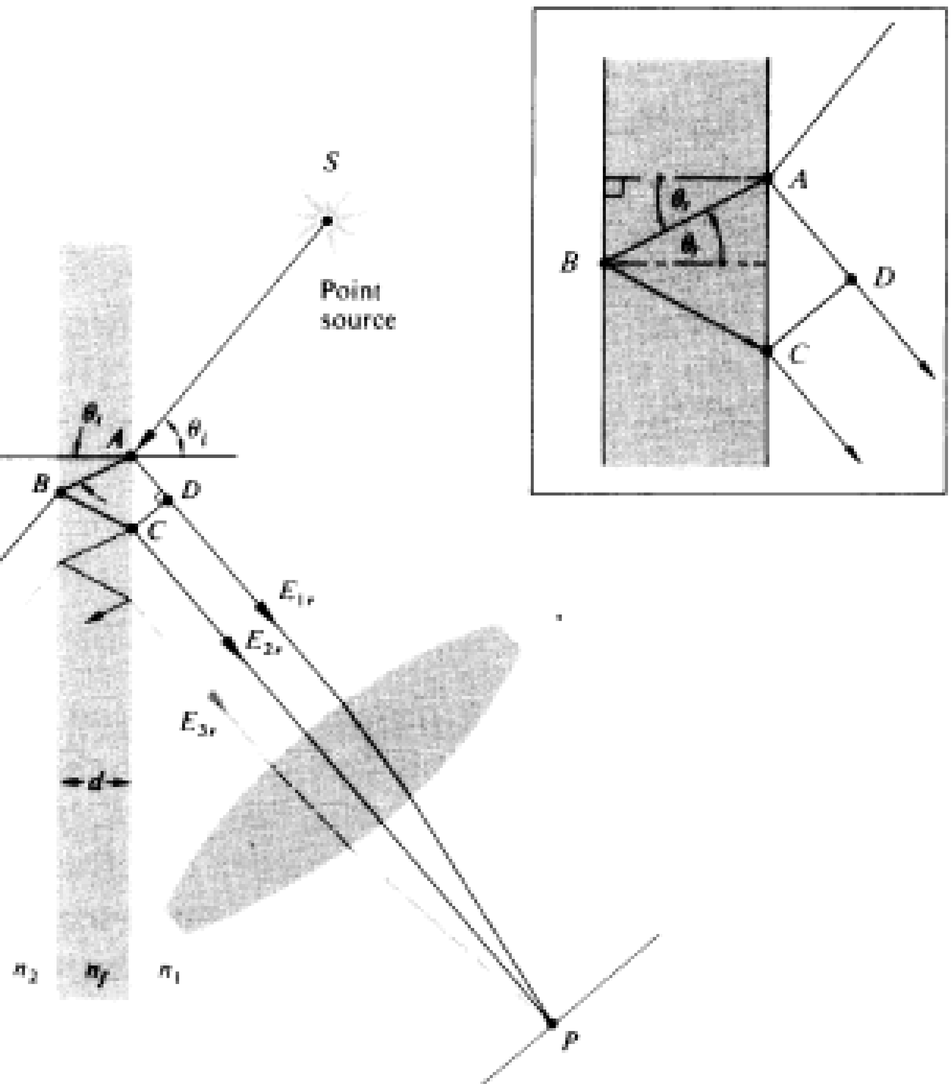


9.4.1 Dielectric Films – Double-Beam Interference





Dielectric Film



From Fig. 9.17, the optical path length difference for the first two reflected beams is given by

$$\Lambda = n_f[(\overline{AB}) + (\overline{BC})] - n_1(\overline{AD})$$

and since $(\overline{AB}) = (\overline{BC}) = d/\cos\theta_t$,

$$\Lambda = \frac{2n_f d}{\cos\theta_t} - n_1(\overline{AD})$$

Now, to find an expression for $(\overline{AD})$, write

$$(\overline{AD}) = (\overline{AC})\sin\theta_i$$

Using Snell's Law, this becomes

$$(\overline{AD}) = (\overline{AC})\frac{n_f}{n_1}\sin\theta_t$$

where

$$(\overline{AC}) = 2d\tan\theta_t \qquad (9.32)$$

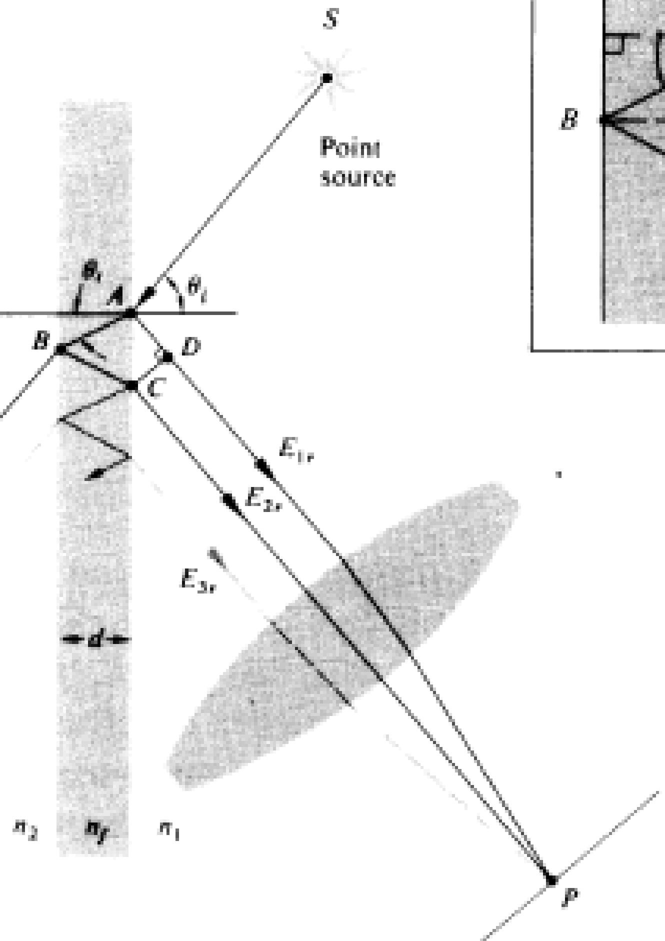
The expression for Λ now becomes

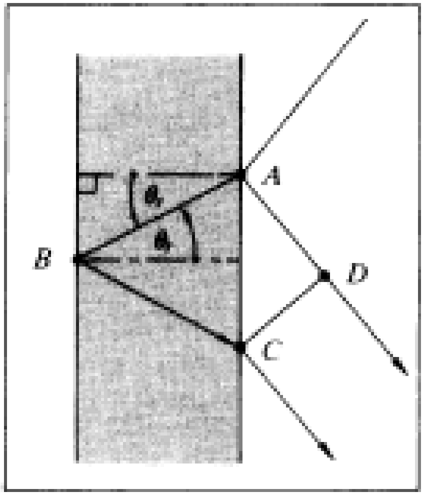
$$\Lambda = \frac{2n_f d}{\cos\theta_t}(1 - \sin^2\theta_t)$$

or finally

$$\Lambda = 2n_f d\cos\theta_t \qquad (9.33)$$

Dielectric Film





$$\delta = k_0 \Lambda \pm \pi$$

and more explicitly

$$\delta = \frac{4\pi n_f}{\lambda_0} d \cos \theta_t \pm \pi \tag{9.34}$$

or

$$\delta = \frac{4\pi d}{\lambda_0} (n_f^2 - n^2 \sin^2 \theta_i)^{1/2} \pm \pi \tag{9.35}$$

The sign of the phase shift is immaterial, so we will choose the negative sign to make the equations a bit simpler. In reflected light an interference maximum, a bright spot, appears at P when $\delta = 2m\pi$—in other words, an even multiple of π. In that case Eq. (9.34) can be rearranged to yield

[maxima]

$$d \cos \theta_t = (2m + 1) \frac{\lambda_f}{4} \tag{9.36}$$

where $m = 0, 1, 2, \ldots$ and use has been made of the fact that $\lambda_f = \lambda_0 / n_f$. This also corresponds to minima in the transmitted light. Interference minima in reflected light (maxima in transmitted light) result when $\delta = (2m \pm 1)\pi$, that is, odd multiples of π. For such cases Eq. (9.34) yields

[minima]

$$d \cos \theta_t = 2m \frac{\lambda_f}{4} \tag{9.37}$$

[maxima] $$d \cos \theta_t = (2m + 1) \frac{\lambda_f}{4} \qquad (9.36)$$

[minima] $$d \cos \theta_t = 2m \frac{\lambda_f}{4} \qquad (9.37)$$

- Fringes of Equal Inclination (θ -- Haidinger fringes)

- Fringes of Equal Thickness (d – Netwon fringes)

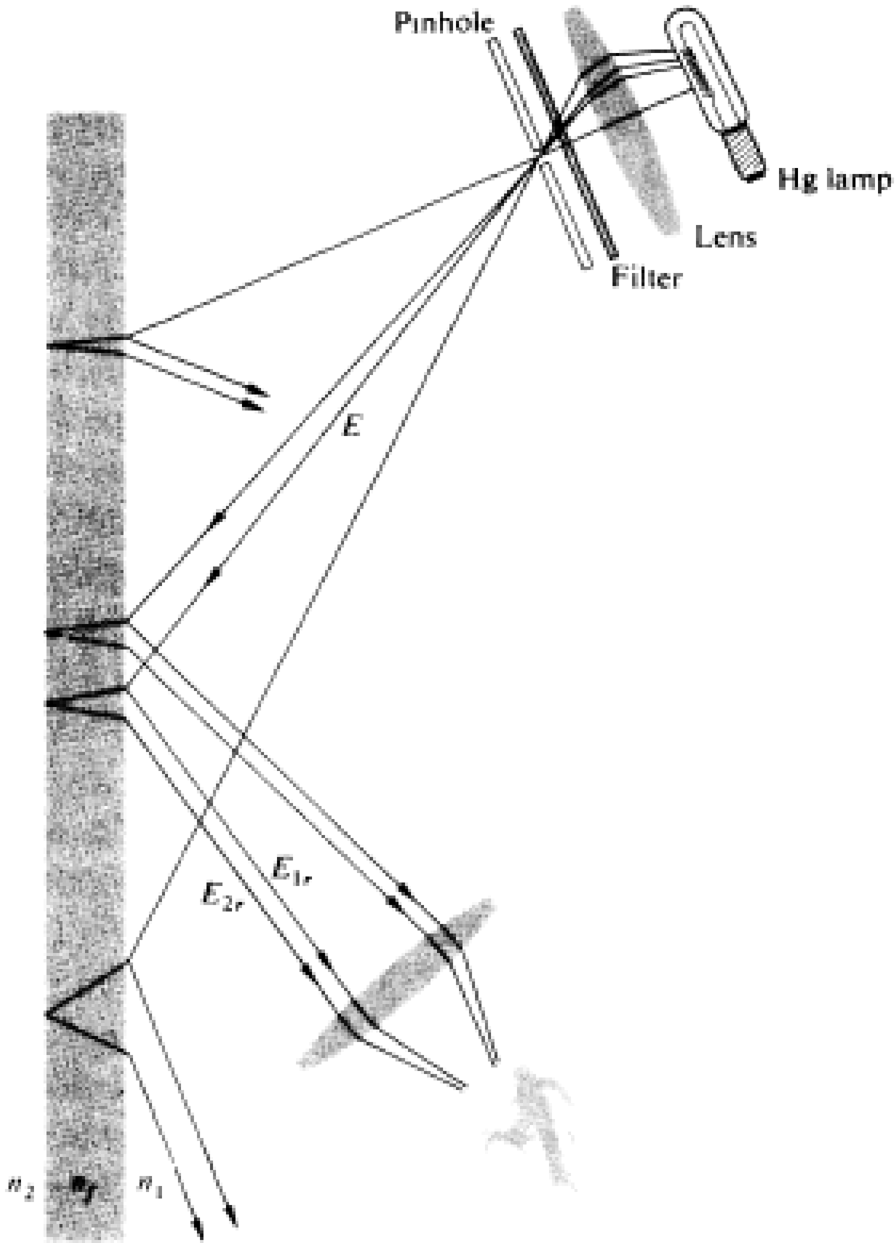
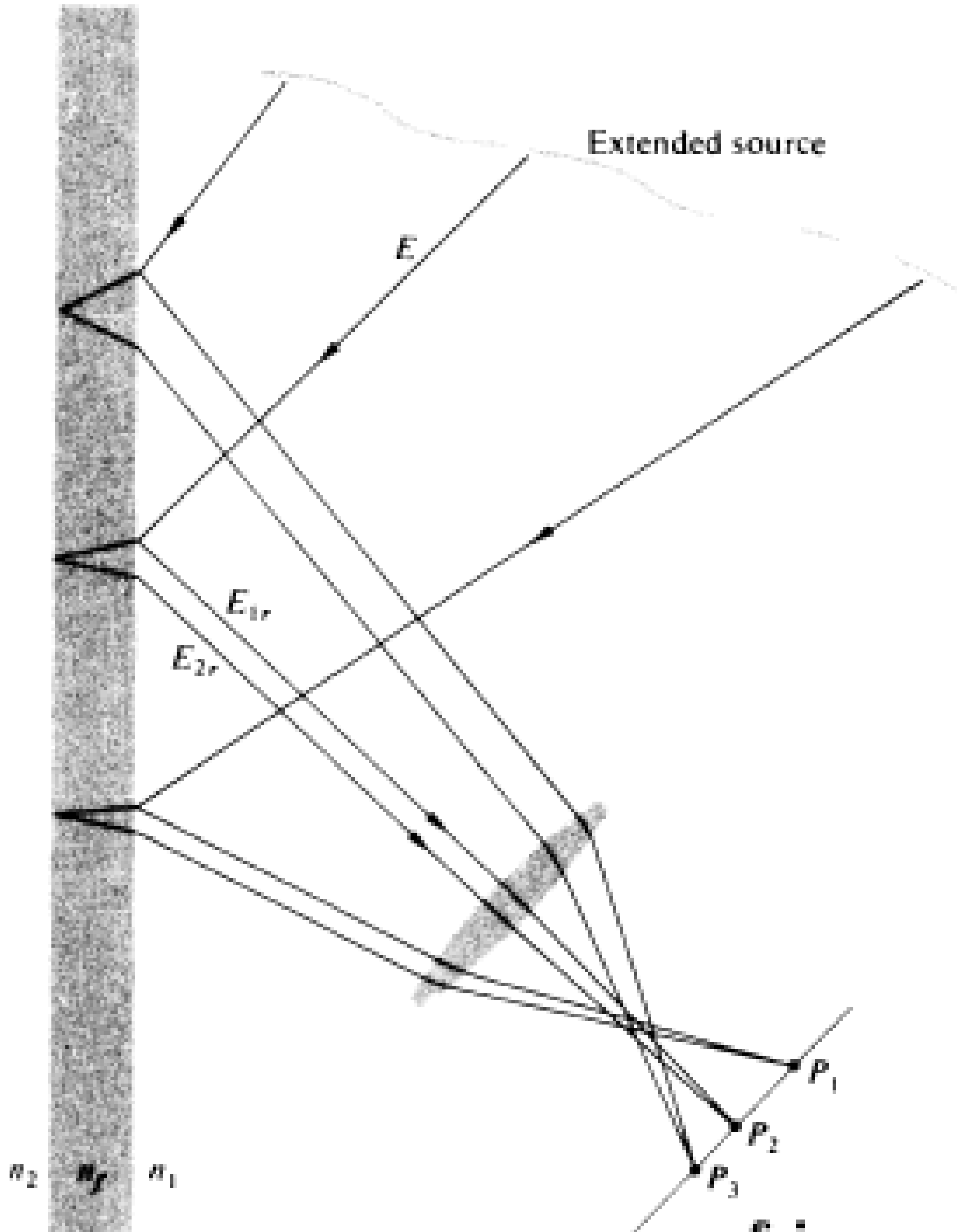


Figure 9.18 Fringes seen on a small portion of the film.



fringes of equal inclination

Figure 9.19 Fringes seen on a large region of the film.

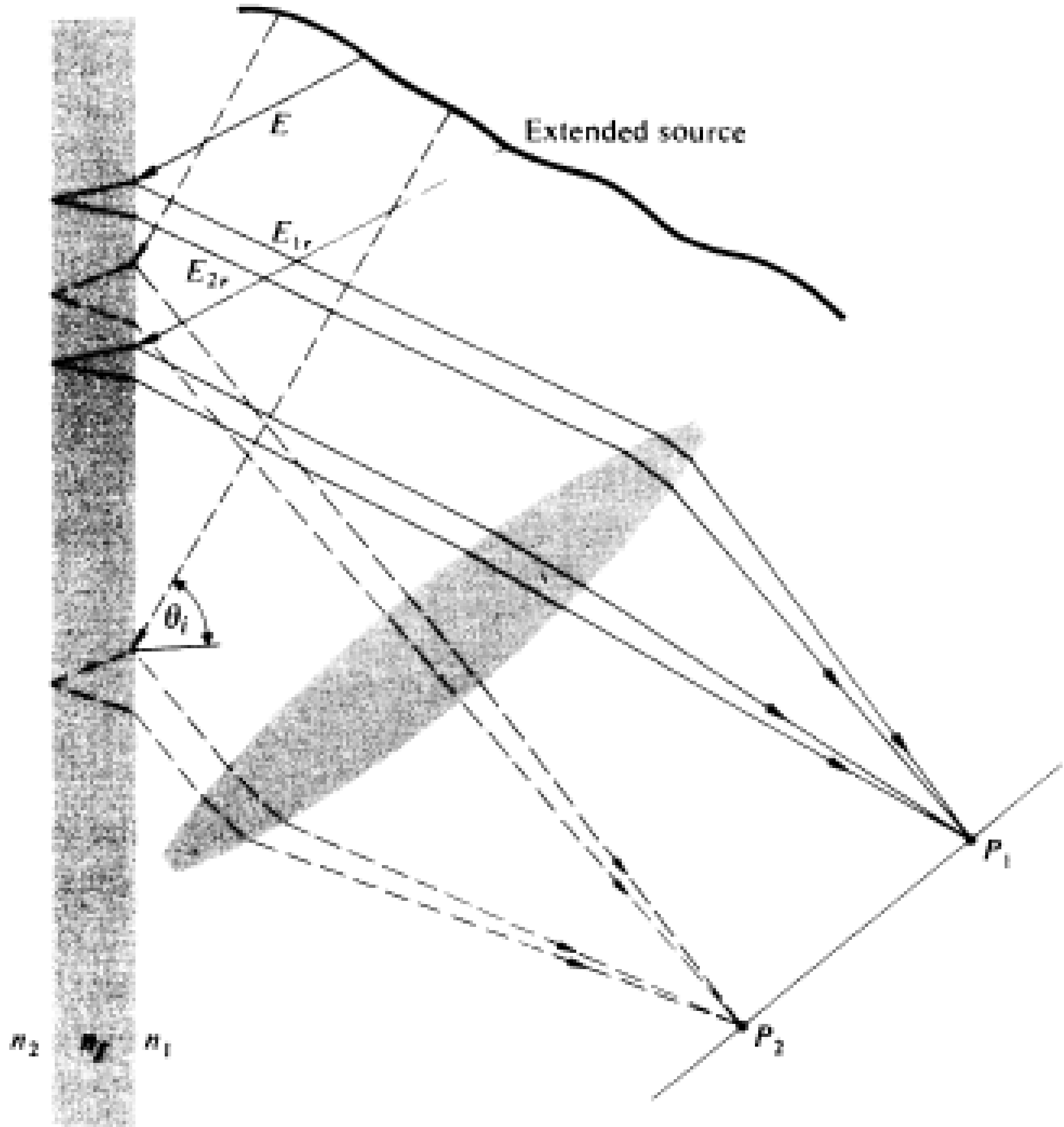
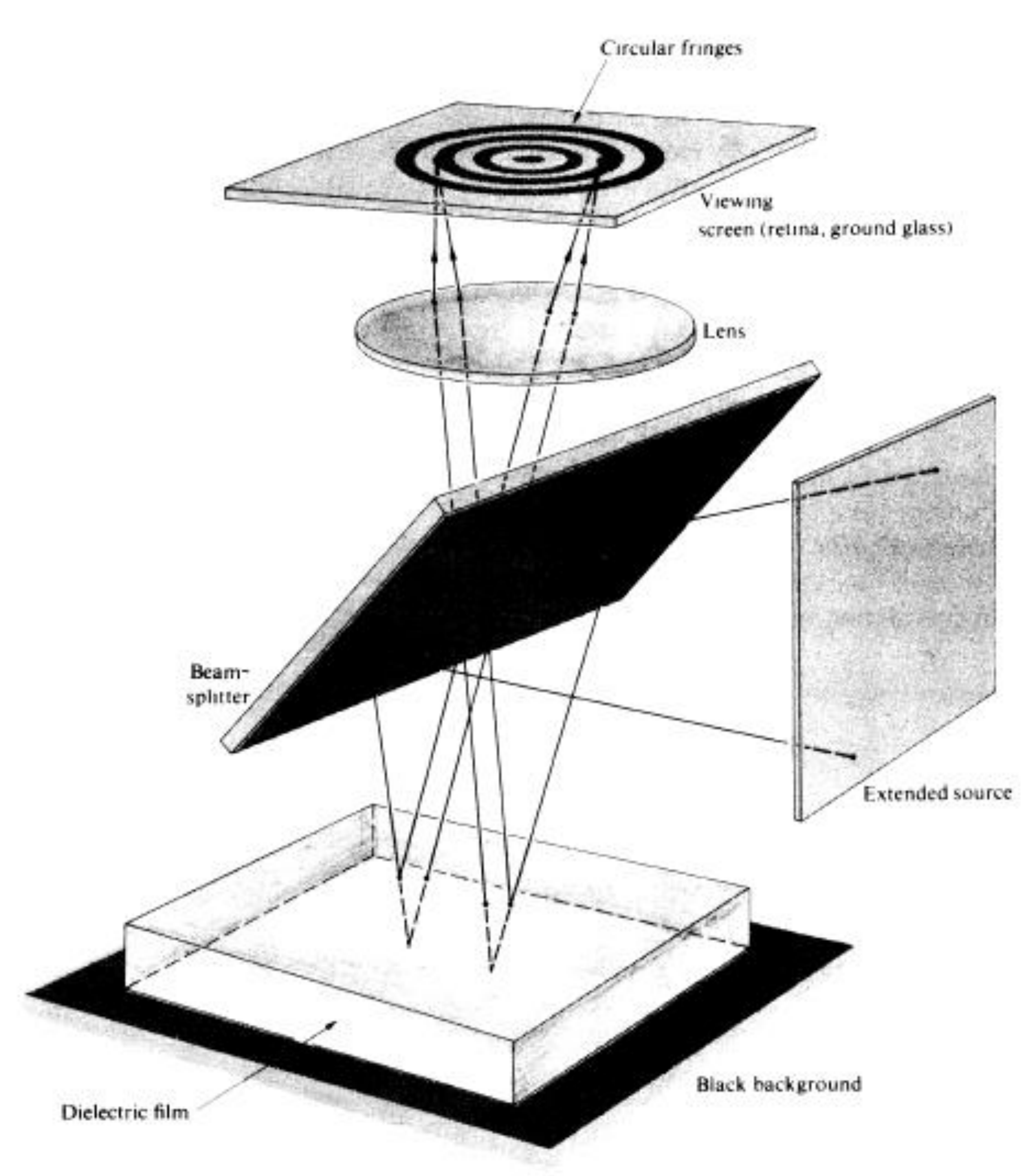


Figure 9.20 All rays inclined at the same angle arrive at the same point.

Haidinger Fringes



Fringes of Equal Thickness

Fizeau fringes

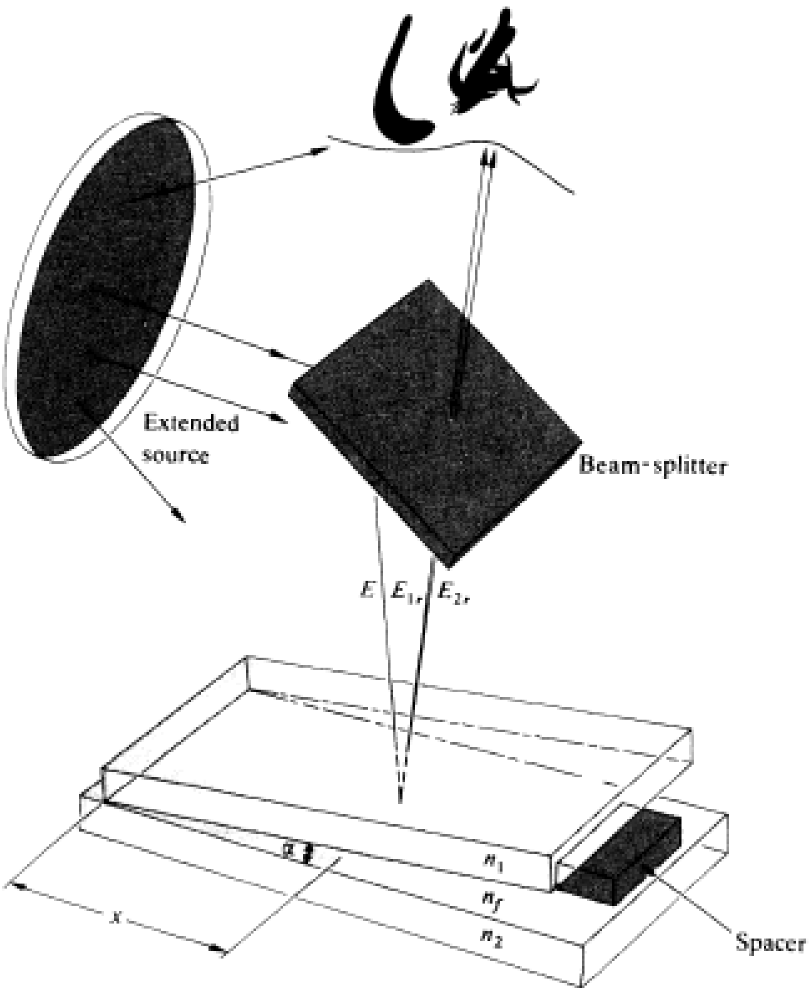


Figure 9.22 Fringes from a wedge-shaped film.

$$d = x\alpha \tag{9.38}$$

For small values of θ_i the condition for an interference maximum becomes

$$(m + \tfrac{1}{2})\lambda_0 = 2n_f d_m$$

or

$$(m + \tfrac{1}{2})\lambda_0 = 2\alpha x_m n_f$$

Since $n_f = \lambda_0/\lambda_f$, x_m may be written as

$$x_m = \left(\frac{m + 1/2}{2\alpha}\right)\lambda_f \tag{9.39}$$

Maxima occur at distances from the apex given by $\lambda_f/4\alpha$, $3\lambda_f/4\alpha$, and so on, and consecutive fringes are separated by a distance Δx, given by

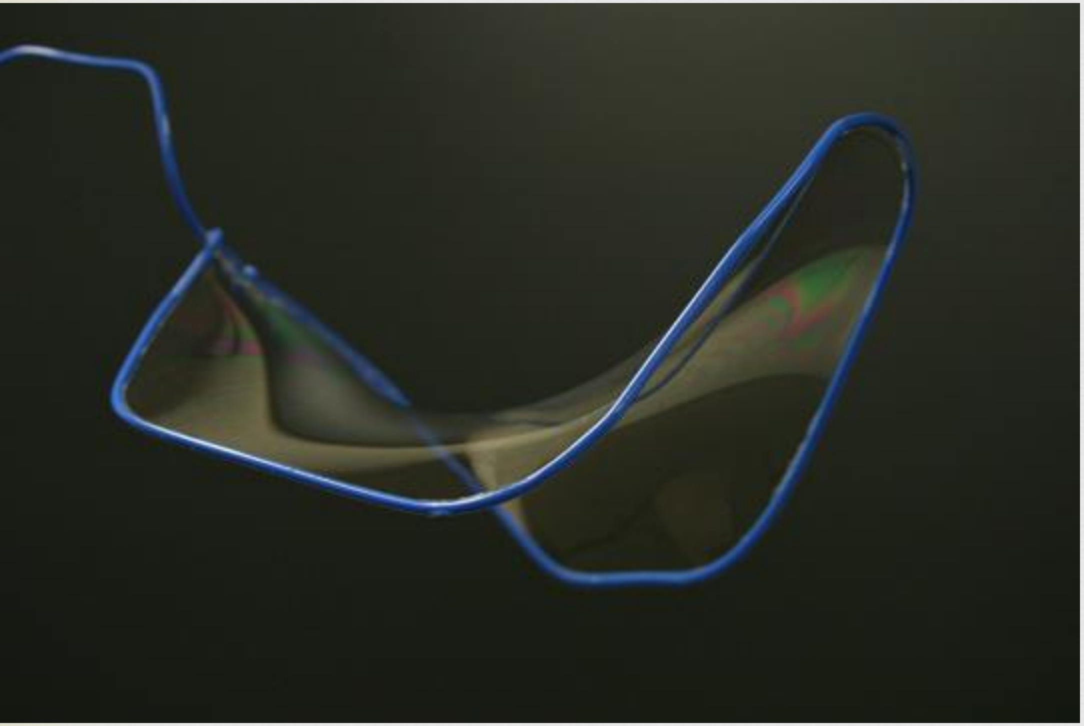
$$\Delta x = \lambda_f/2\alpha \tag{9.40}$$

Notice that the difference in film thickness between adjacent maxima is simply $\lambda_f/2$. Since the beam reflected from the lower surface traverses the film twice ($\theta_i \approx \theta_t \approx 0$), adjacent maxima differ in optical path length by λ_f. Note, too, that the film thickness at the various maxima is given by

$$d_m = (m + \tfrac{1}{2})\frac{\lambda_f}{2} \tag{9.41}$$

which is an odd multiple of a quarter wavelength. Traversing the film twice yields a phase shift of π, which, when added to the shift of π resulting from reflection, puts the two rays back in-phase.

Soap Films



https://www.youtube.com/watch?v=4I34jA1fDp4

Newton's Rings

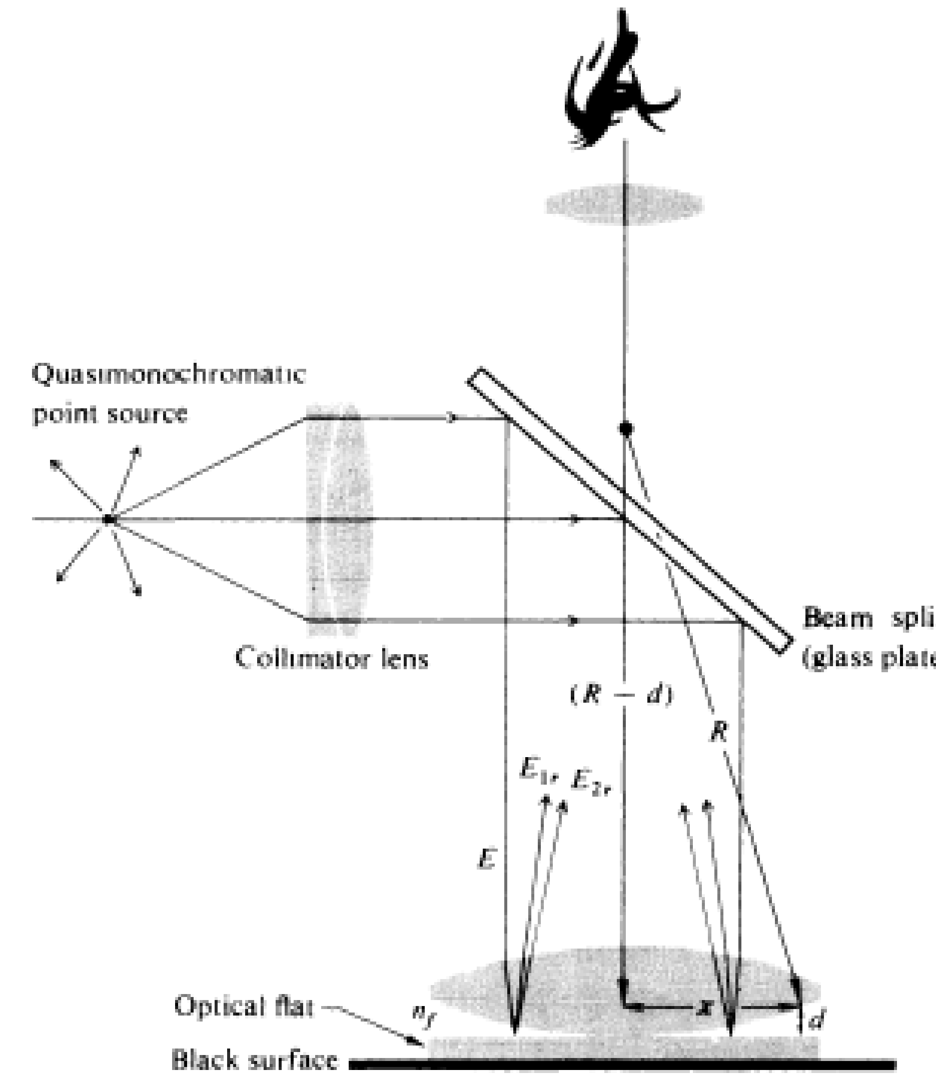


Figure 9.23 A standard setup to observe Newton's rings