PHYS 3038 Optics
L13 Interference
Reading Material: Ch9.1-2

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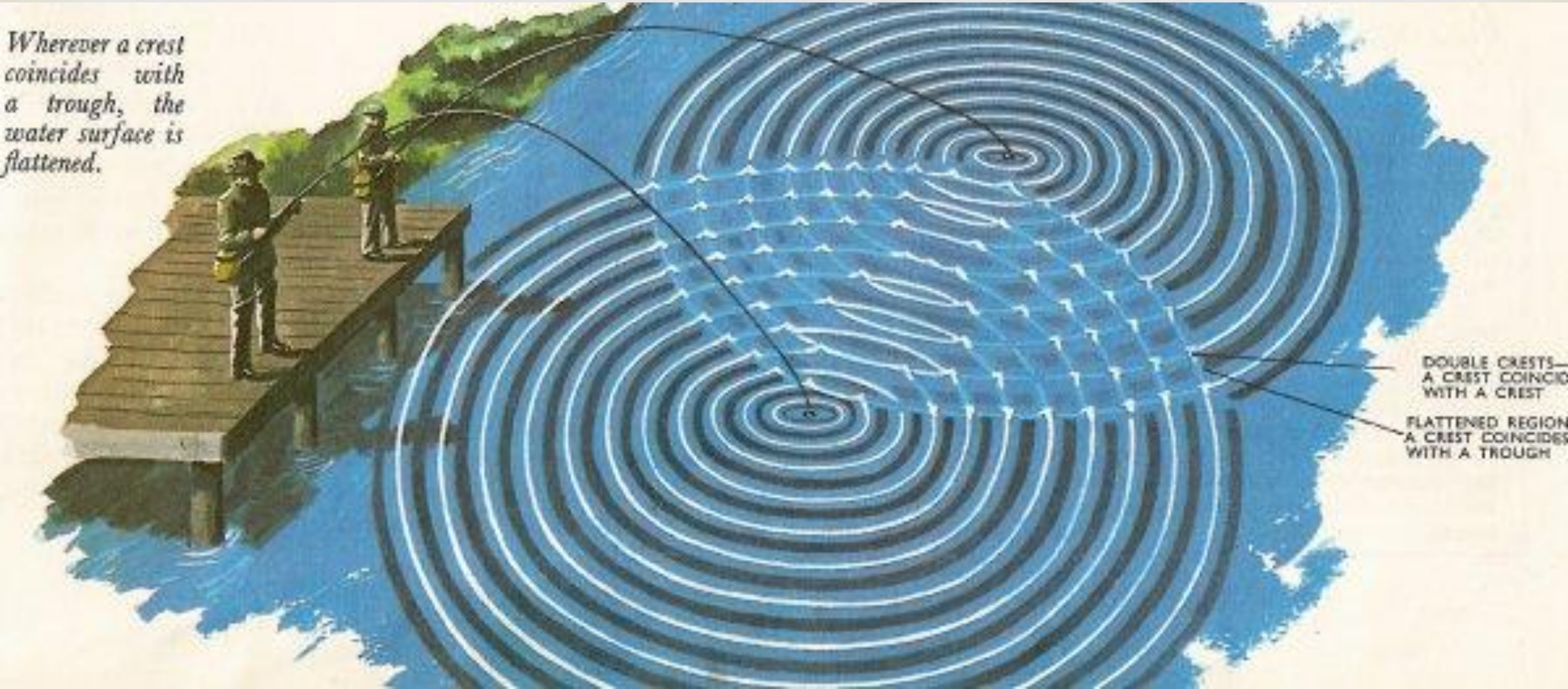
Interference of the waves between the same frequency



Interferences

$$\vec{E} = \sum_m \vec{E}_m$$

$$\vec{E}_m = \vec{E}_{0m} e^{i(\vec{k}_m \cdot \vec{r} - \omega t + \varepsilon_m)}$$

$$I = |\vec{E}|^2 = \left|\sum_m \vec{E}_i\right|^2 = \vec{E}^* \cdot \vec{E} = \left(\sum_m \vec{E}_m^*\right) \cdot \left(\sum_m \vec{E}_m\right)$$

$$\neq \sum_m |\vec{E}_i|^2 = \sum_m (\vec{E}_m^* \cdot \vec{E}_i)$$

Two-Wave Interference

$$\vec{E}_1 = \vec{E}_{01} e^{i(\vec{k}_1 \cdot \vec{r} - \omega t + \varepsilon_1)}$$

$\vec{E}_{01}$ and $\vec{E}_{02}$ are both real

$$\vec{E}_2 = \vec{E}_{02} e^{i(\vec{k}_2 \cdot \vec{r} - \omega t + \varepsilon_1)}$$

$$I = \left|\vec{E}\right|^2 = \left|\vec{E}_1 + \vec{E}_2\right|^2 = \left(\vec{E}_1 + \vec{E}_2\right)^* \cdot \left(\vec{E}_1 + \vec{E}_2\right)$$

$$= \vec{E}_1^* \cdot \vec{E}_1 + \vec{E}_2^* \cdot \vec{E}_2 + \vec{E}_1^* \cdot \vec{E}_2 + \vec{E}_2^* \cdot \vec{E}_1$$

$$= \left|\vec{E}_1\right|^2 + \left|\vec{E}_2\right|^2 + 2Re\{\vec{E}_1^* \cdot \vec{E}_2\}$$

$$= \left|\vec{E}_{01}\right|^2 + \left|\vec{E}_{02}\right|^2 + 2Re\{\vec{E}_{01} \cdot \vec{E}_{02} e^{i\left[\left(\vec{k}_2 - \vec{k}_1\right) \cdot \vec{r} + (\varepsilon_2 - \varepsilon_1)\right]}\}$$

$$= \left|\vec{E}_{01}\right|^2 + \left|\vec{E}_{02}\right|^2 + 2\left(\vec{E}_{01} \cdot \vec{E}_{02}\right) \cos\left[\left(\vec{k}_2 - \vec{k}_1\right) \cdot \vec{r} + (\varepsilon_2 - \varepsilon_1)\right]$$

$\vec{E}_1 \,||\, \vec{E}_2$

$$I = \left|\vec{E}_{01}\right|^2 + \left|\vec{E}_{02}\right|^2 + 2\left(\vec{E}_{01} \cdot \vec{E}_{02}\right)\cos\left[\left(\vec{k}_2 - \vec{k}_1\right) \cdot \vec{r} + (\varepsilon_2 - \varepsilon_1)\right]$$

$$\delta = \left(\vec{k}_2 - \vec{k}_1\right) \cdot \vec{r} + (\varepsilon_2 - \varepsilon_1)$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \qquad (9.15)$$

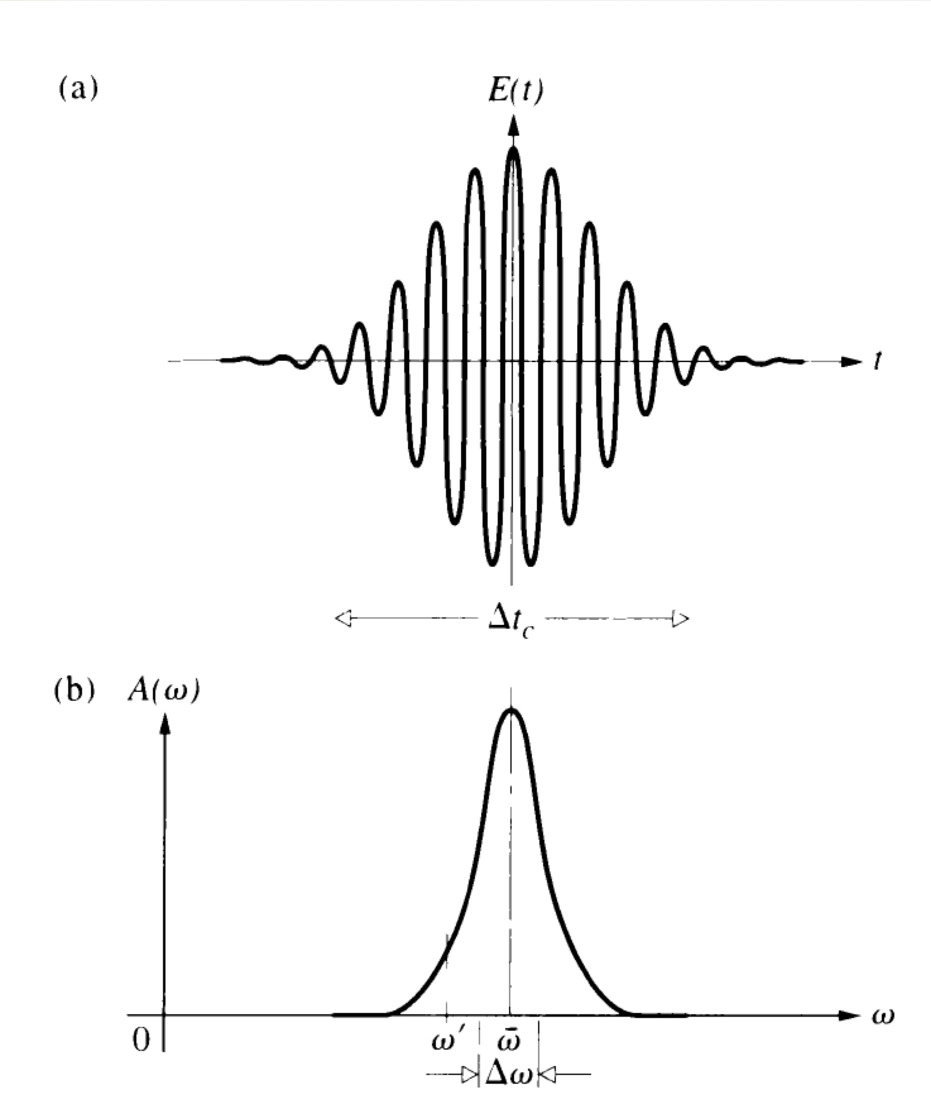
when $\delta = 0, \pm 2\pi, \pm 4\pi, \ldots$

In this case of ***total constructive interference***, the phase dif-

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \qquad (9.16)$$

This occurs when $\delta = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$, and it is referred to as ***total destructive interference***.

Temporal & Spatial Coherence



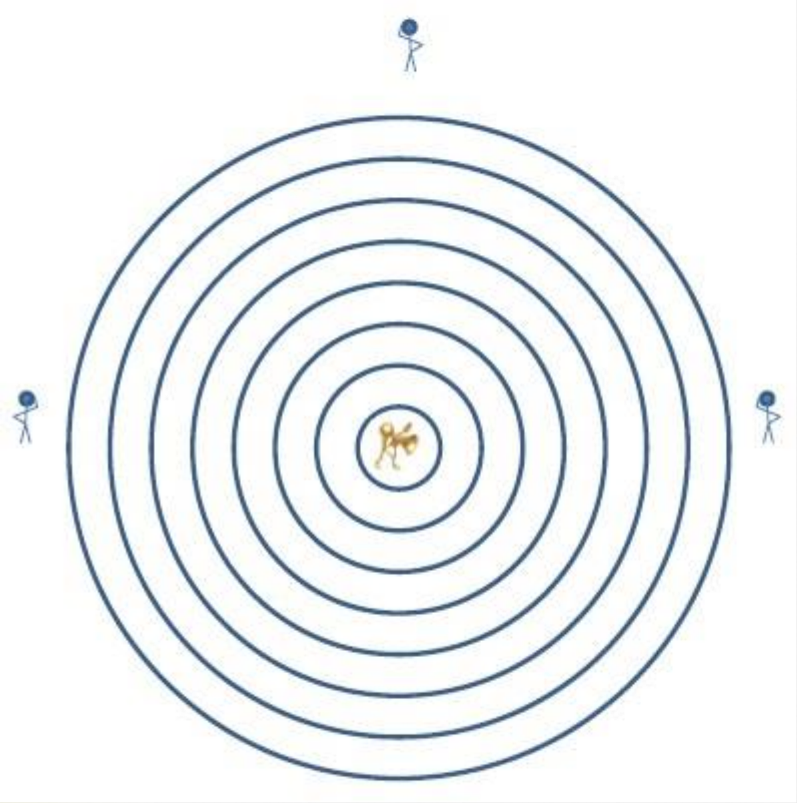
$$E(z,t) = \int A(\omega) e^{i[k(\omega)z-\omega t]} d\omega$$

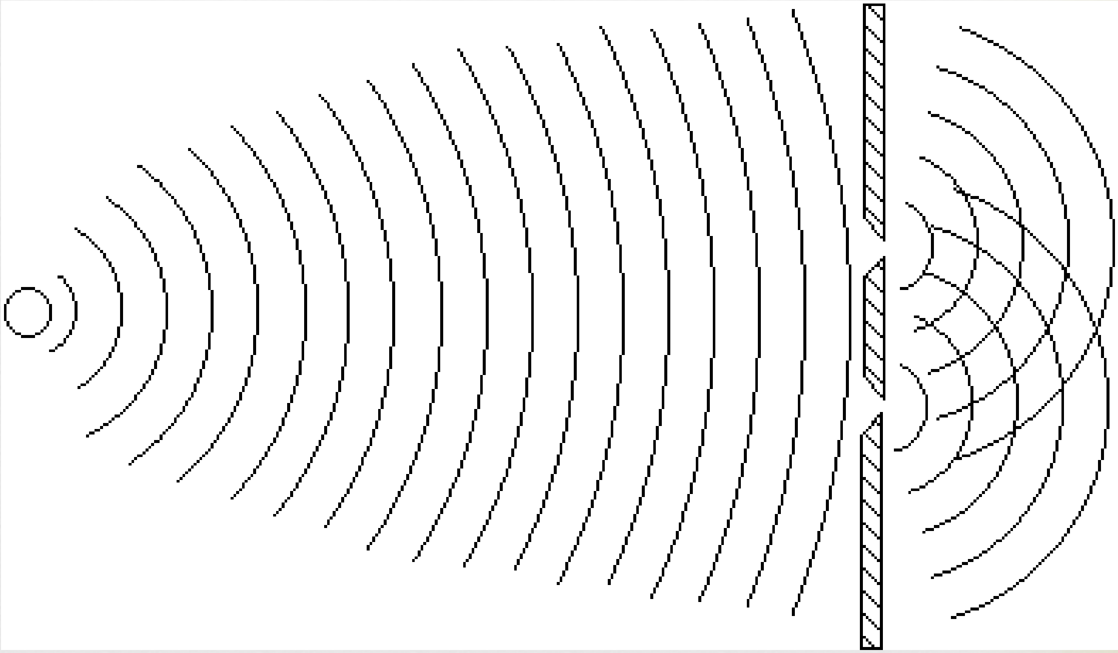
Bandwidth (FWHM) $\Delta\omega = 2\pi\Delta\nu$

Coherence time $\Delta t_c = 1/\Delta\nu$

Coherence length $\Delta l_c = c\Delta t_c$

9.3 Wavefront-Splitting Interferometers





9.3 Wavefront-Splitting Interferometers
Young's Experiment

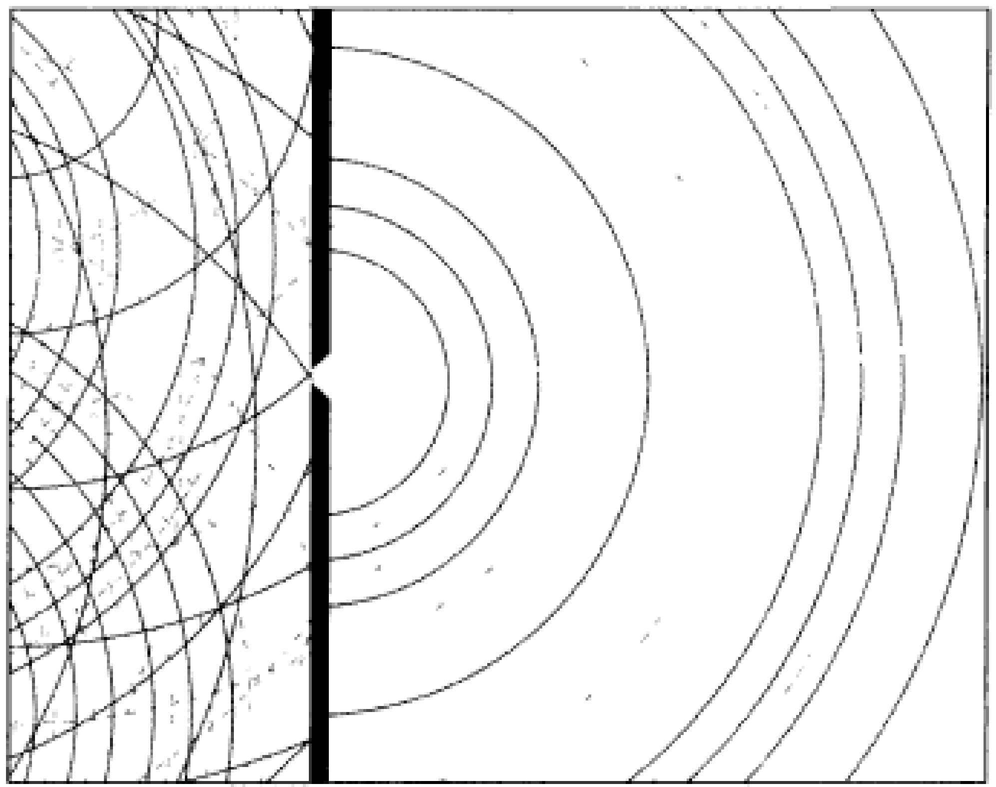
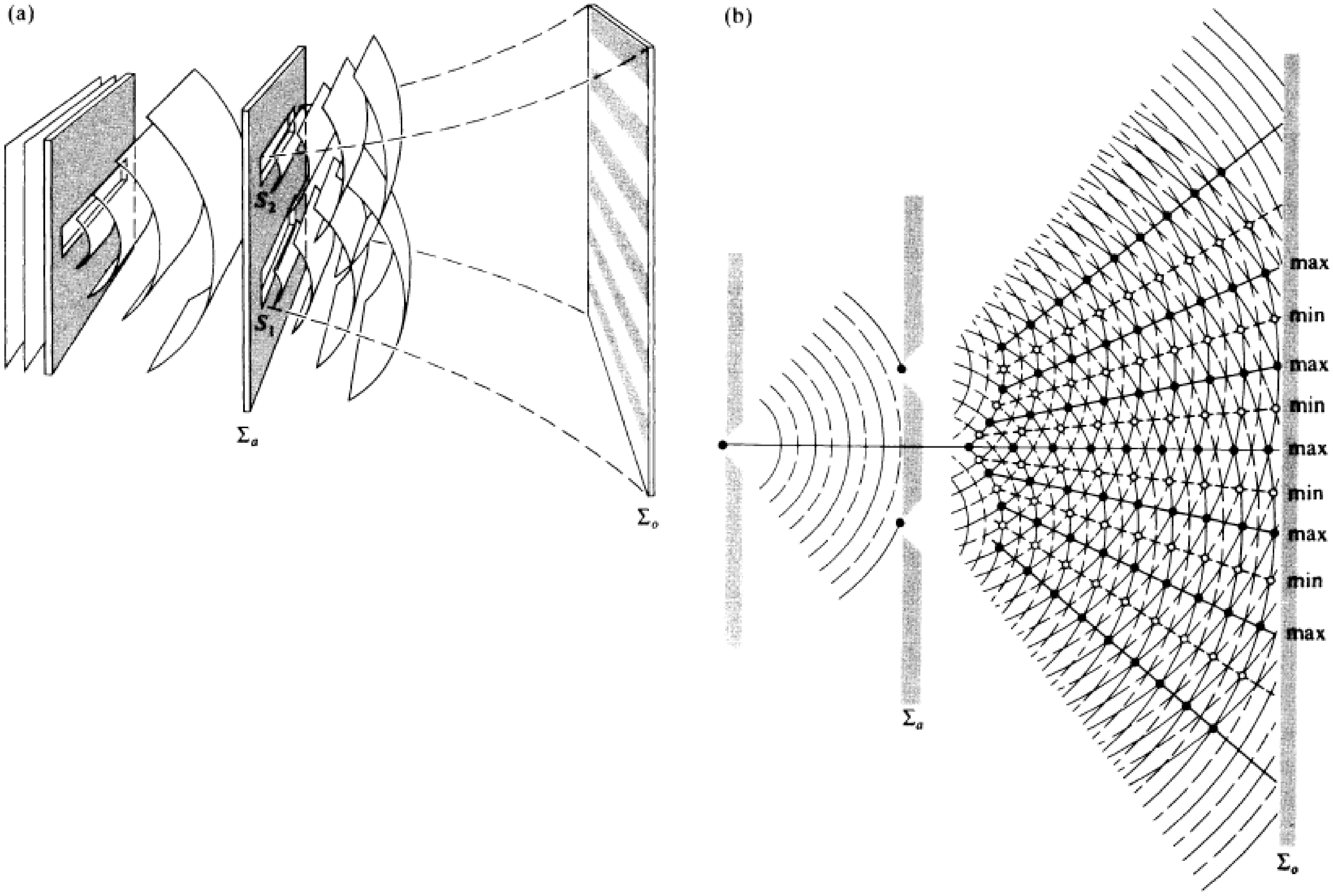
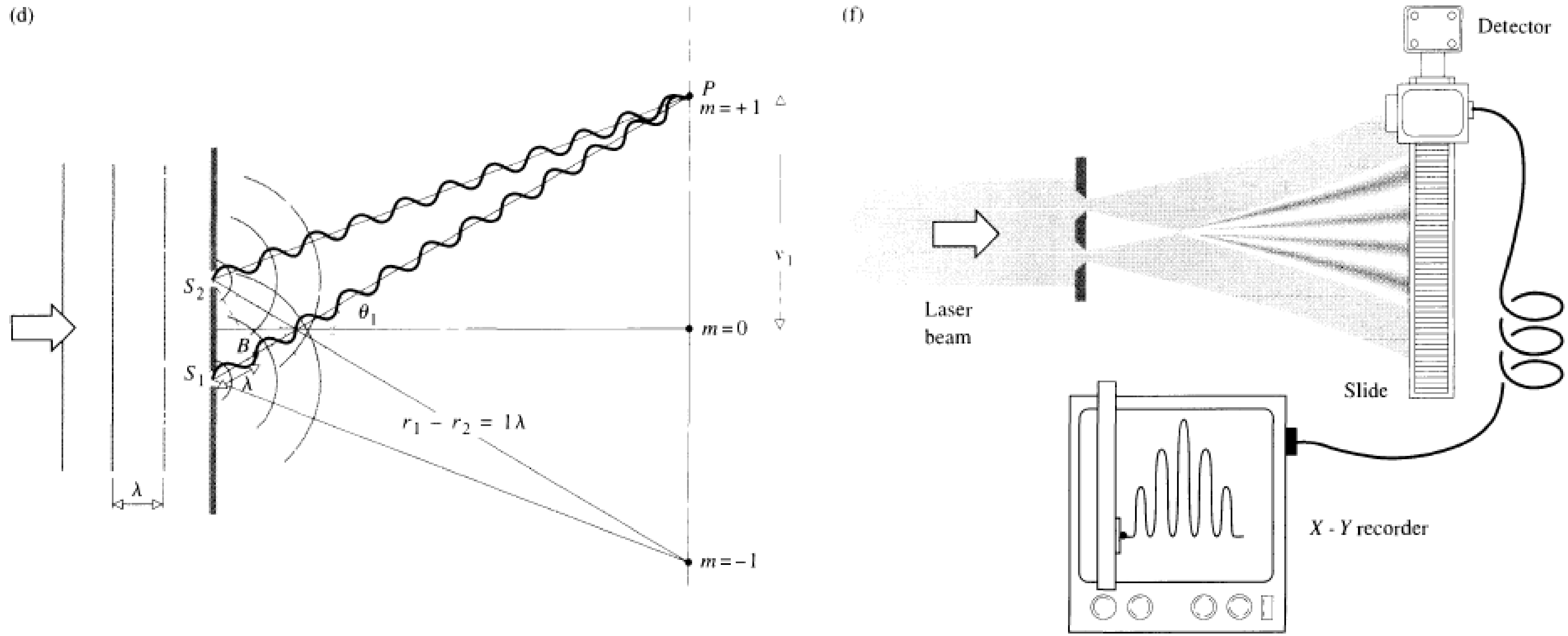


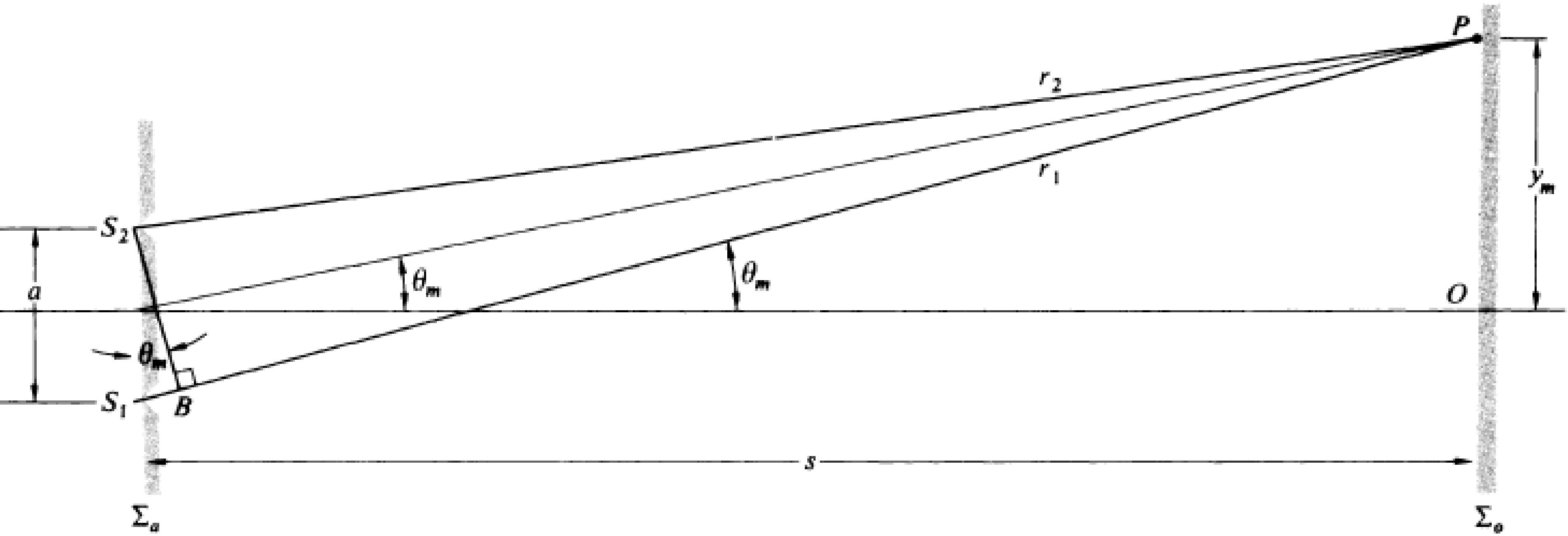
Figure 9.7 The pinhole scatters a wave that is spatially coherent, even though it's not temporally coherent.

Young's Experiment



Young's Experiment





$$(\overline{S_1B}) = (\overline{S_1P}) - (\overline{S_2P}) \tag{9.22}$$

or
$$(\overline{S_1B}) = r_1 - r_2$$

Continuing with this approximation (see Problem 9.15), $(r_1 - r_2) = a \sin\theta$ and so

$$r_1 - r_2 \approx a\theta \tag{9.23}$$

since $\theta \approx \sin\theta$. Notice that

$$\theta \approx \frac{y}{s} \tag{9.24}$$

$$\delta = k(r_1 - r_2)$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

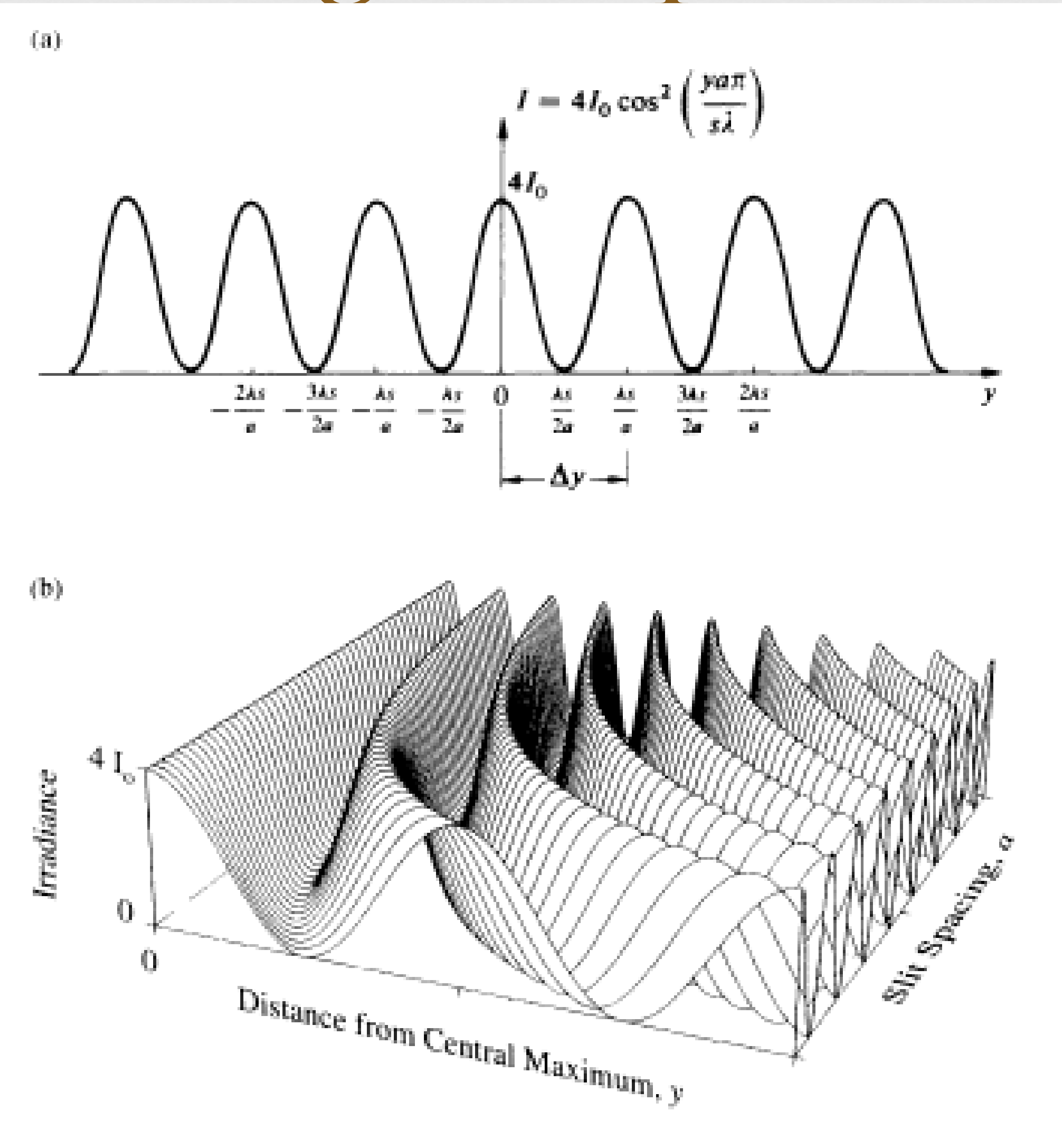
$$I_1 = I_2 = I_0$$

$$I = 2I_0 + 2I_0\cos\delta$$

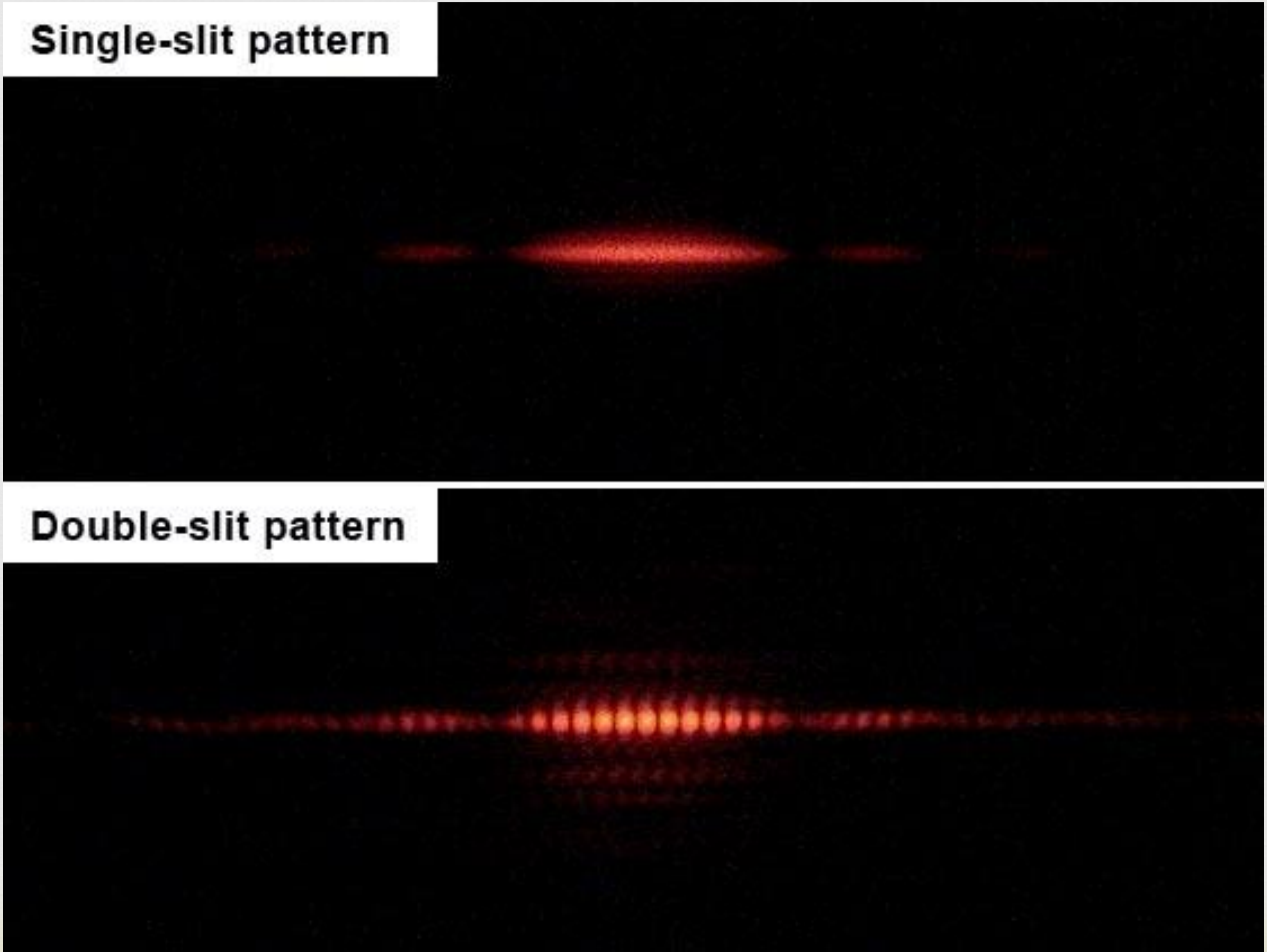
$$= 4I_0\cos^2\delta/2$$

$$= 4I_0\cos^2\frac{kay}{2s} = 4I_0\cos^2\frac{\pi ay}{\lambda s}$$

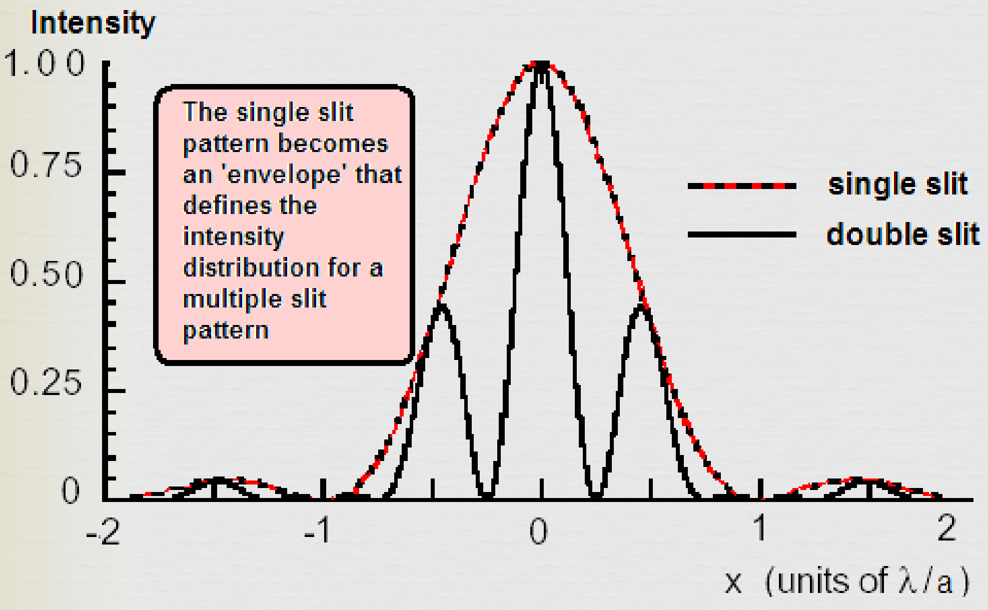
Young's Experiment

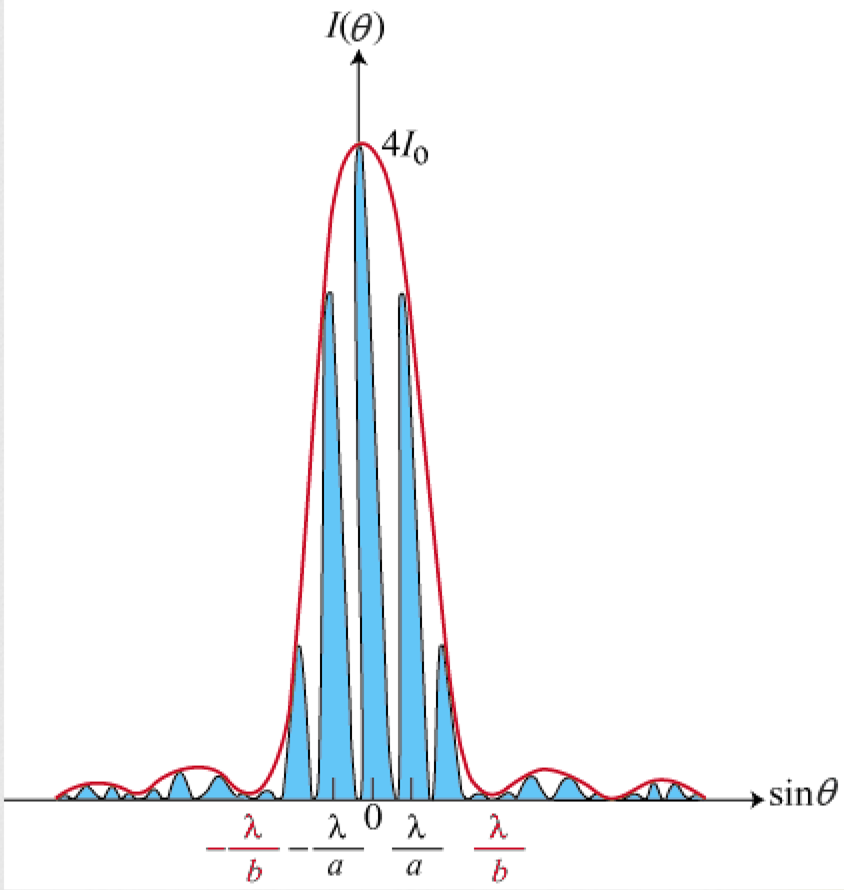


Young's Experiment

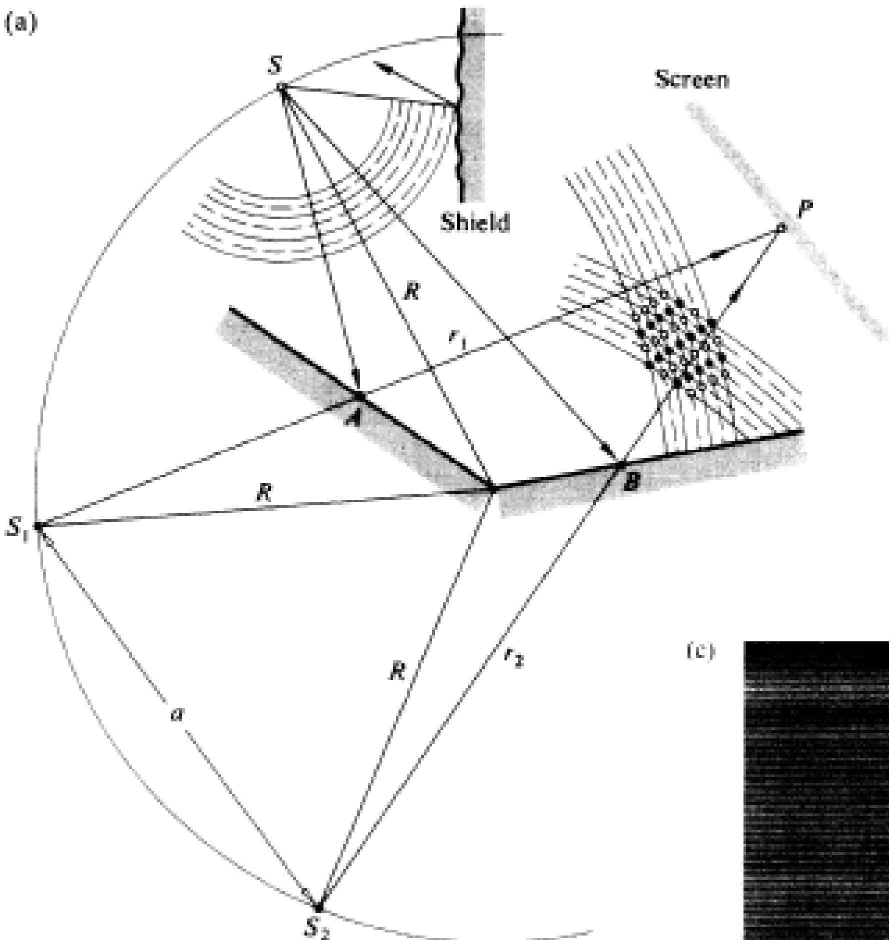


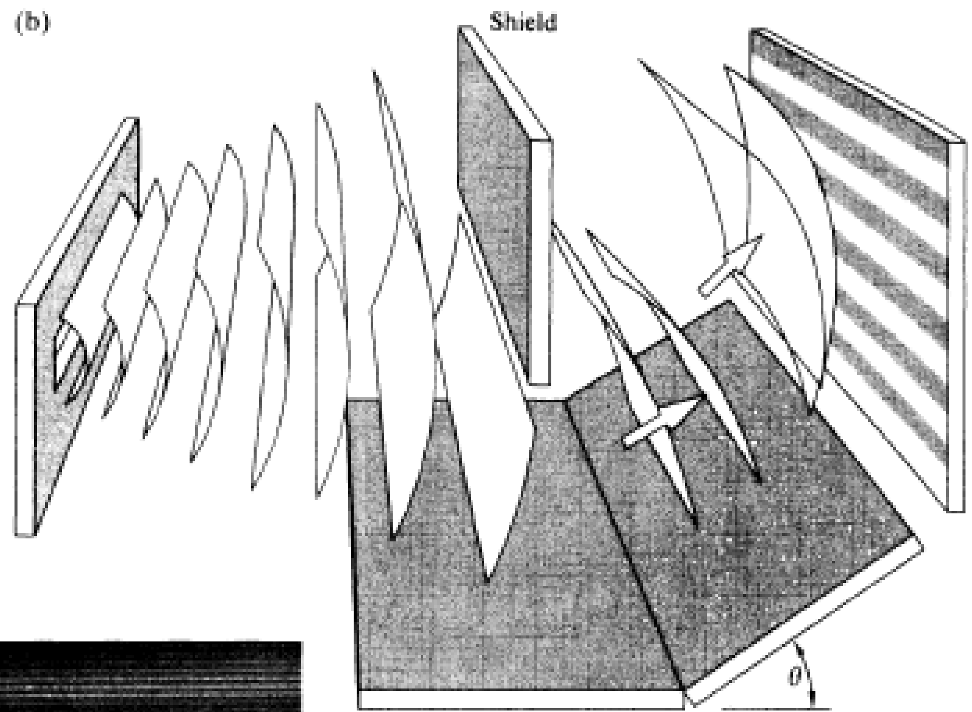
Young's Experiment





Fresnel's Double Mirror





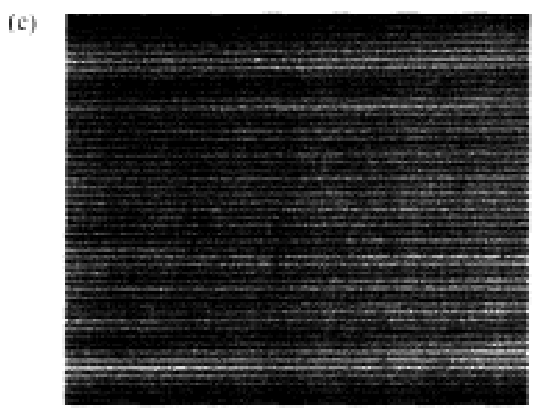


Figure 9.12 (*a*) Fresnel's double mirror. (*b*) Two waves, one reflected from each mirror, interfere. (*c*) These fringes were obtained at a wavelength of only 13.9 nm using radiation from the LURE synchrotron at Orsay, France. (Photo courtesy D. Joyeux, *Institut d' Optique*.)

Fresnel's Biprism

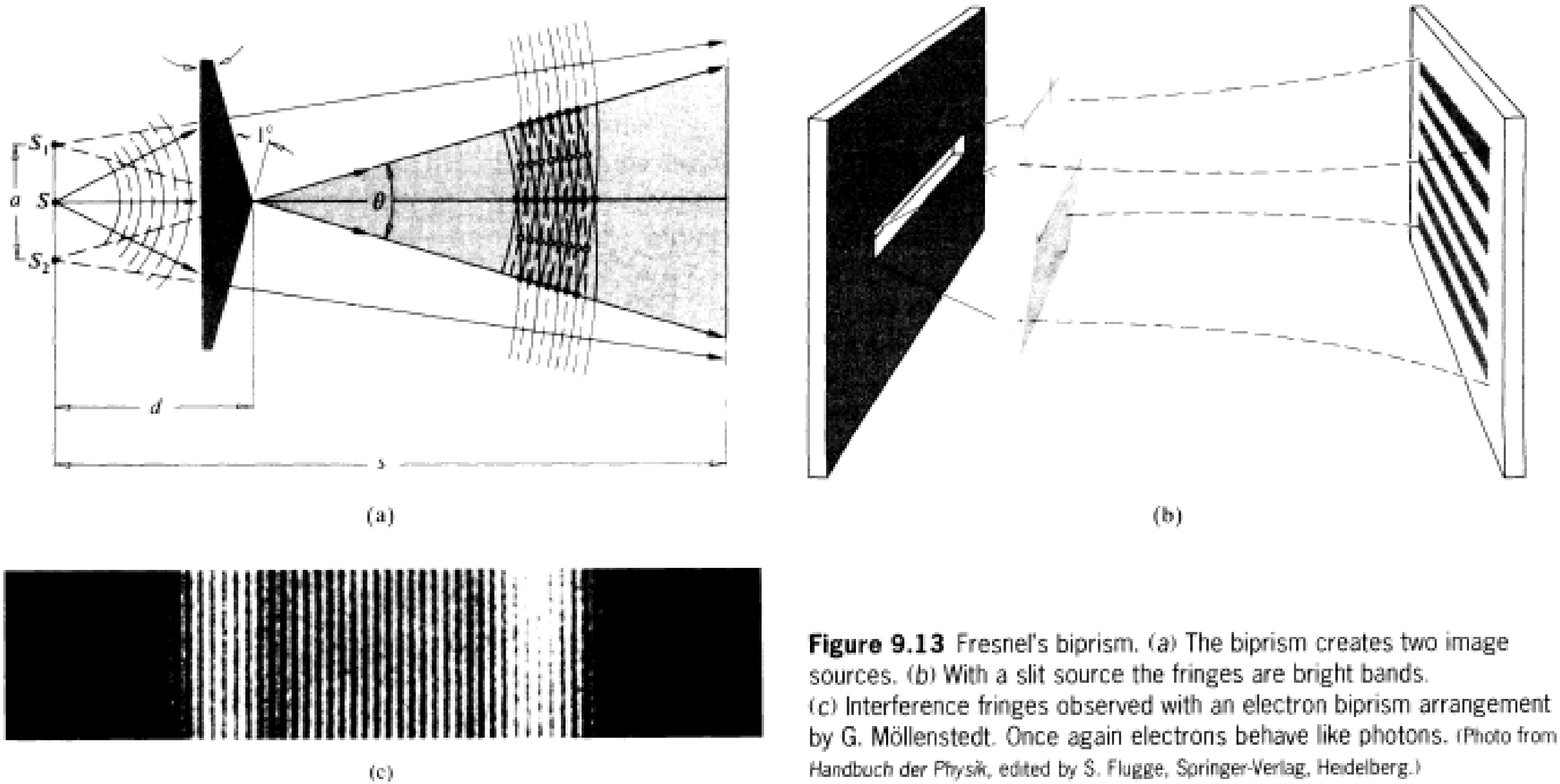


Figure 9.13 Fresnel's biprism. (*a*) The biprism creates two image sources. (*b*) With a slit source the fringes are bright bands. (*c*) Interference fringes observed with an electron biprism arrangement by G. Möllenstedt. Once again electrons behave like photons. (Photo from *Handbuch der Physik*, edited by S. Flugge, Springer-Verlag, Heidelberg.)

Lloyd’s Mirror

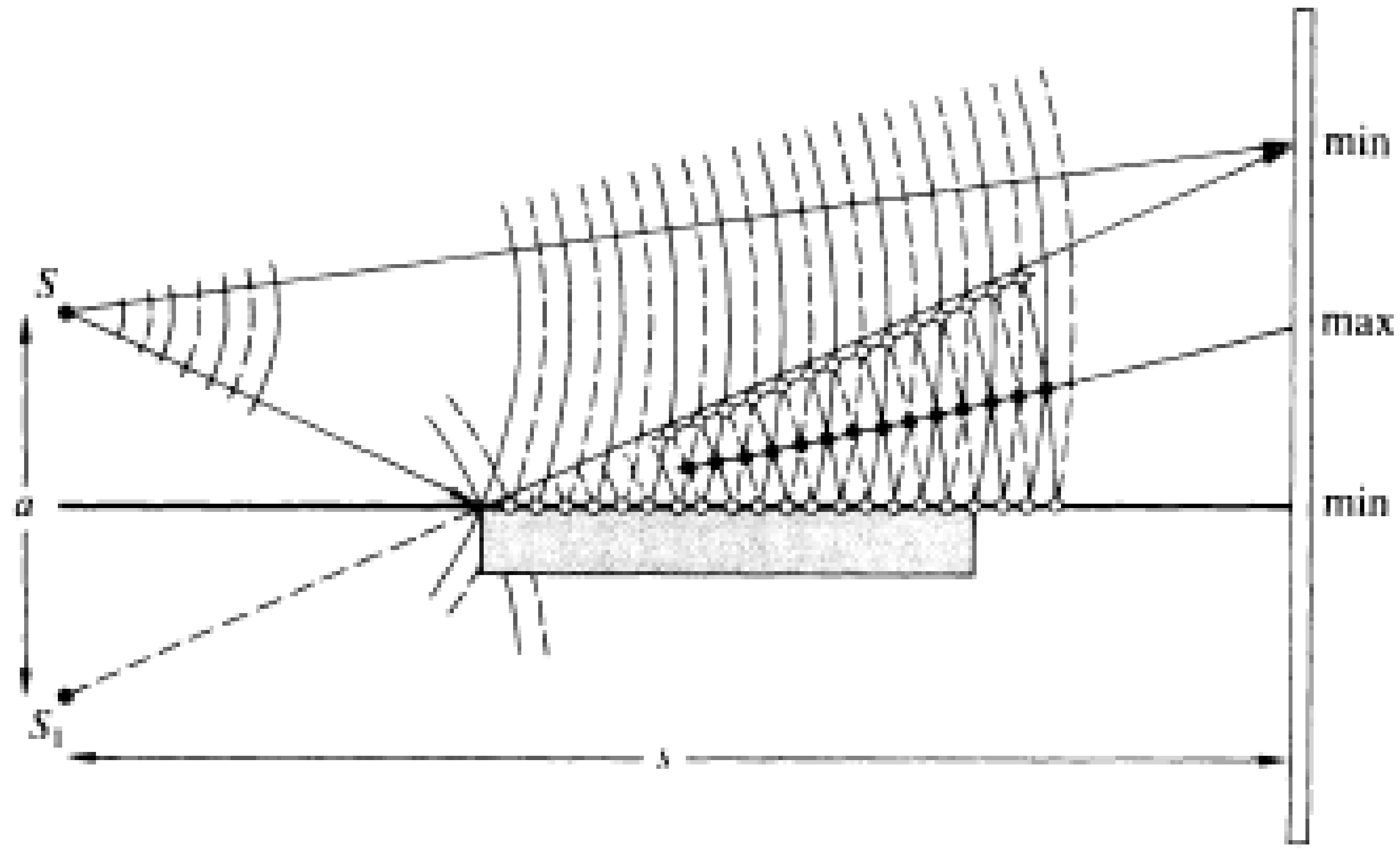


Figure 9.14 Lloyd's mirror.