PHYS 3038 Optics
L11 Polarization
Reading Material: Ch8

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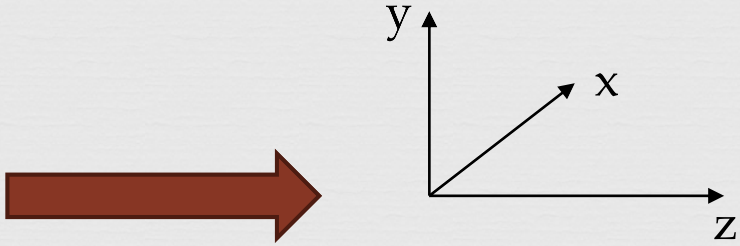
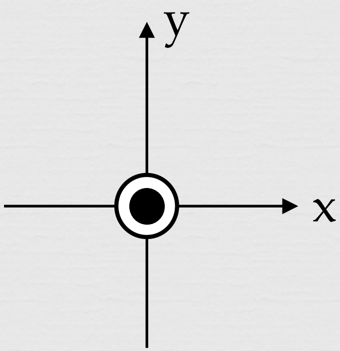
2015, the Year of Light

E Vector field





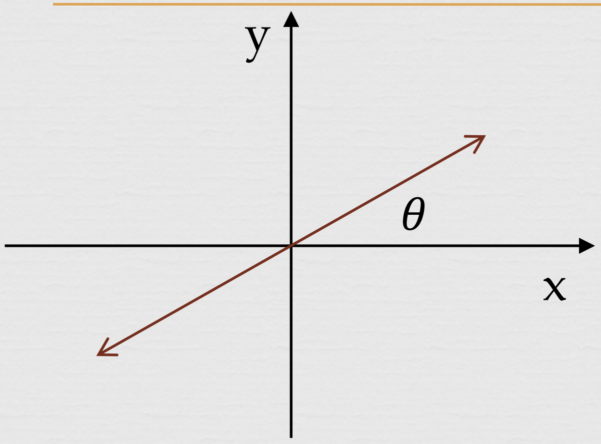
Convention of the xyz coordinator system: Look toward the light source

$$\vec{\mathbf{E}}(z,t) = \vec{\mathbf{E}}_x(z,t) + \vec{\mathbf{E}}_y(z,t)$$

$$\vec{\mathbf{E}}_x(z,t) = \hat{\mathbf{i}}\, E_{0x} \cos\,(kz - \omega t) \quad = \hat{\imath} E_{0x} \cos(\omega t - kz)$$

$$\vec{\mathbf{E}}_y(z,t) = \hat{\mathbf{j}}\, E_{0y} \cos\,(kz - \omega t + \varepsilon) \quad = \hat{\jmath} E_{0y} \cos(\omega t - kz - \varepsilon)$$

Linear Polarization $\varepsilon = 0$



$$\vec{E}_x = \hat{\imath} E_{0x} \cos(\omega t - kz)$$

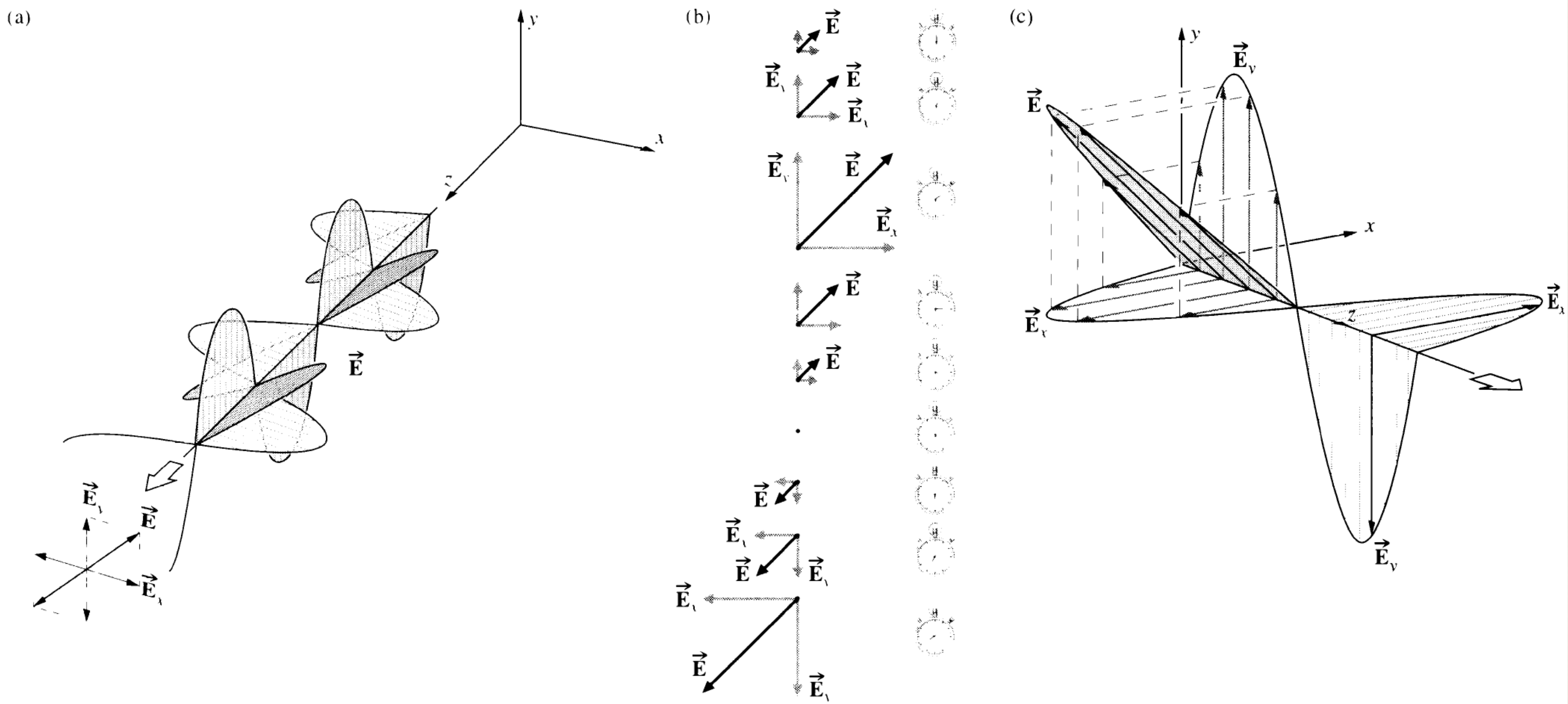
$$\vec{E}_y = \hat{\jmath} E_{0y} \cos(\omega t - kz)$$

$$E_{0x} = E_0 \cos\theta$$

$$E_{0y} = E_0 \sin\theta$$

$$\vec{E} = \vec{E}_x + \vec{E}_y = \left(\hat{\imath} E_{0x} + \hat{\jmath} E_{0y}\right) \cos(\omega t - kz) = E_0 \left(\hat{\imath} \cos\theta + \hat{\jmath} \sin\theta\right) \cos(\omega t - kz)$$

Linear Polarization $\varepsilon = 0$



Right Circular Polarization

$$E_{0x} = E_{0y} = E_0, \ \varepsilon = -\frac{\pi}{2}$$

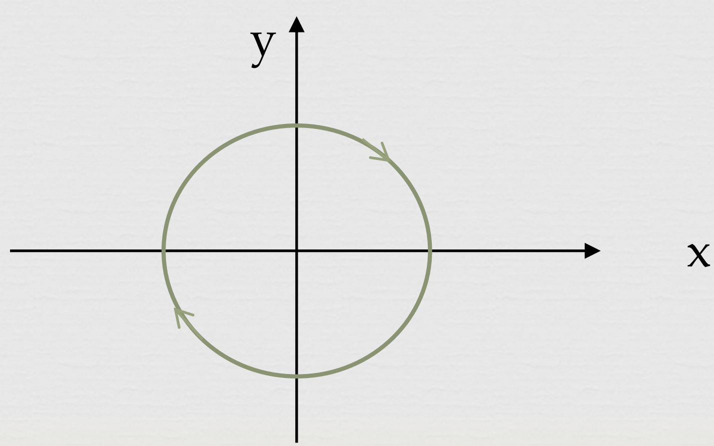
$$E_x = E_{0x}\cos(kz - \omega t)$$

$$E_y = E_{0y}\cos(kz - \omega t + \varepsilon)$$

$$- kz) \xrightarrow{z=0} \hat{\imath}E_0 \cos\omega t$$

$$kz + \frac{\pi}{2}) \xrightarrow{z=0} \hat{\jmath}E_0 \cos\left(\omega t + \frac{\pi}{2}\right) = -\hat{\jmath}E_0 \sin\omega t$$

$$\vec{E}_x \cdot \vec{E}_x + \vec{E}_y \cdot \vec{E}_y = E_0^2$$

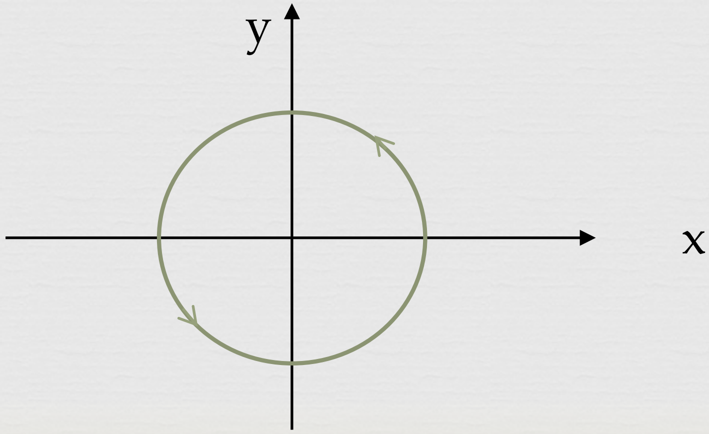


Left Circular Polarization $E_{0x}=E_{0y}=E_0,\ \varepsilon = \frac{\pi}{2}$

$$\vec{E}_x = \hat{\imath}E_0\cos(\omega t - kz) \xrightarrow{z=0} \hat{\imath}E_0\cos\omega t$$

$$\vec{E}_y = \hat{\jmath}E_0\cos(\omega t - kz - \frac{\pi}{2}) \xrightarrow{z=0} \hat{\jmath}E_0\cos\left(\omega t - \frac{\pi}{2}\right) = \hat{\jmath}E_0\sin\omega t$$

$$\vec{E}_x \cdot \vec{E}_x + \vec{E}_y \cdot \vec{E}_y = E_0^2$$



Elliptical Polarization

$E_{0x} \neq E_{0y}$ $\varepsilon \neq 0$

$$E_x = E_{0x} \cos(kz - \omega t)$$

$$E_y = E_{0y} \cos(kz - \omega t + \varepsilon)$$

$$E_y/E_{0y} = \cos(kz - \omega t)\cos\varepsilon - \sin(kz - \omega t)\sin\varepsilon$$

and combine it with E_x/E_{0x} to yield

$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos\varepsilon = -\sin(kz - \omega t)\sin\varepsilon \qquad (8.13)$$

It follows from Eq. (8.11) that

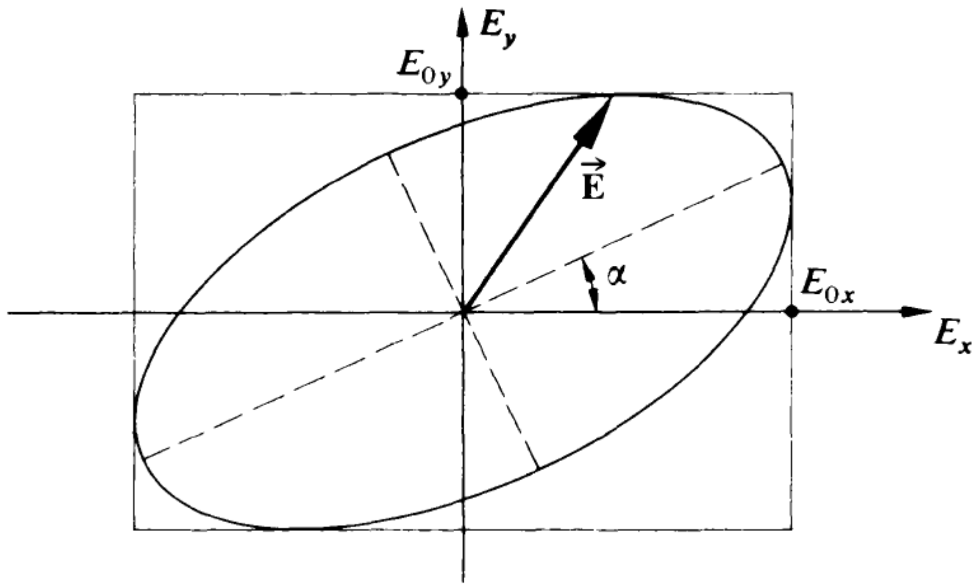
$$\sin(kz - \omega t) = [1 - (E_x/E_{0x})^2]^{1/2}$$

so Eq. (8.13) leads to

$$\left(\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos\varepsilon\right)^2 = \left[1 - \left(\frac{E_x}{E_{0x}}\right)^2\right]\sin^2\varepsilon$$

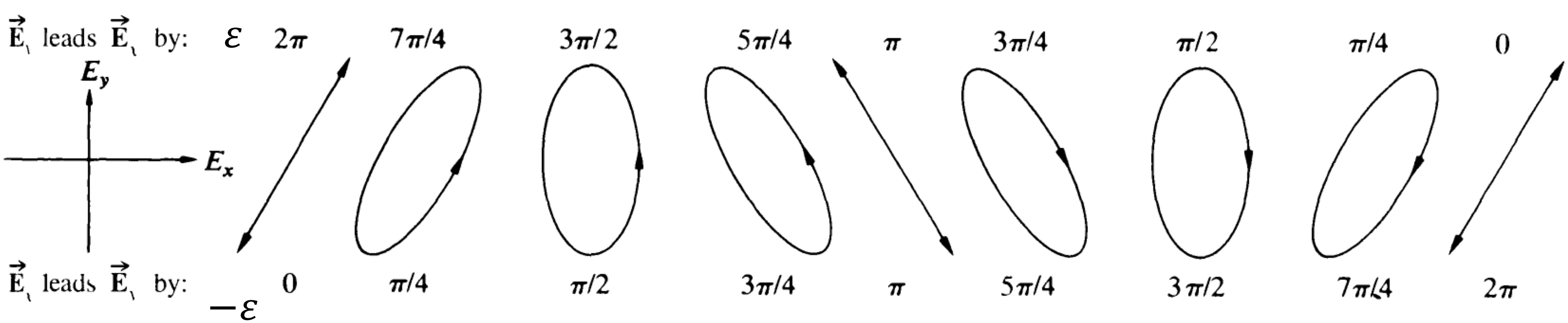
Finally, on rearranging terms, we have

$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\varepsilon = \sin^2\varepsilon \qquad (8.14)$$



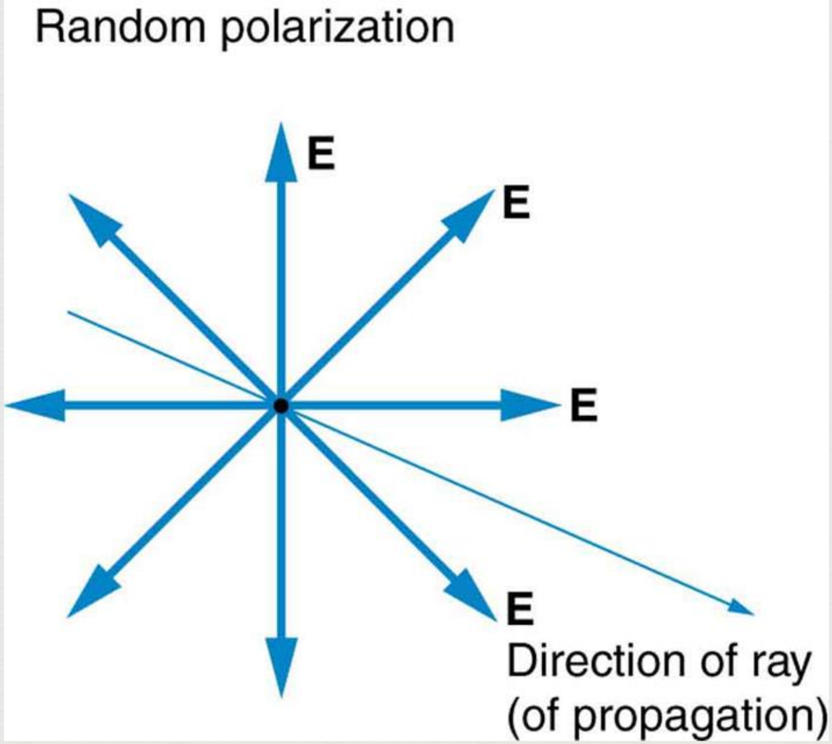
This is the equation of an ellipse making an angle α with the (E_x, E_y)-coordinate system (Fig. 8.6) such that

$$\tan 2\alpha = \frac{2E_{0x}E_{0y}\cos\varepsilon}{E_{0x}^2 - E_{0y}^2} \qquad (8.15)$$


leads by:
ε
2π
7π/4
3π/2
5π/4
π
3π/4
π/2
π/4
0
E_y
E_x
leads by:
−ε
0
π/4
π/2
3π/4
π
5π/4
3π/2
7π/4
2π
(a)

Natural Light

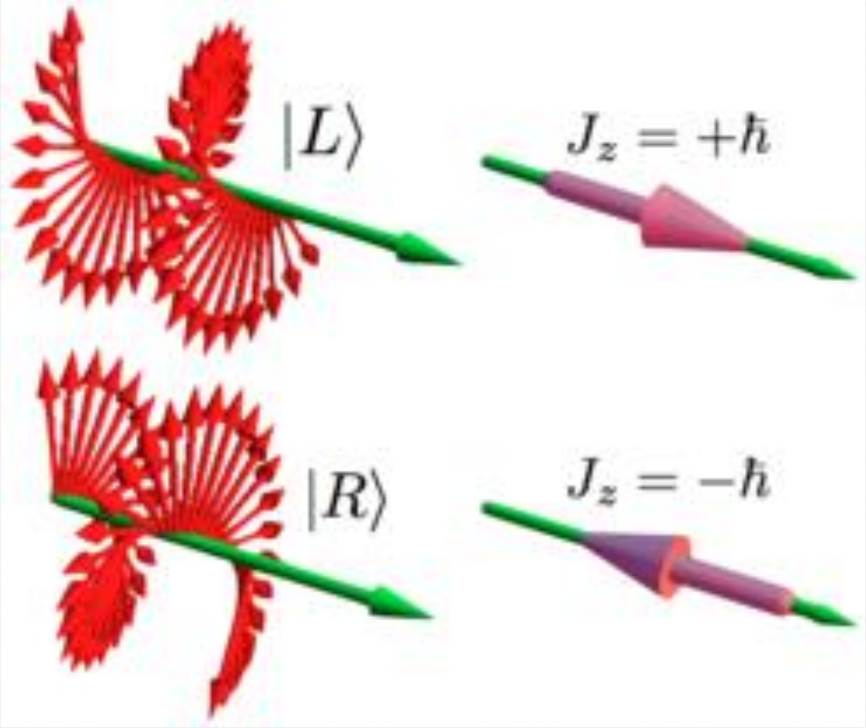
- Unpolarized light: Randomly polarized light



Spin (Angular Momentum) VS Polarization

$S = 1, m_s = -1, 0, 1$

$J_z = -\hbar, 0, \hbar$



Question: Where is $J_z = 0$?

Polarizers

- Linear polarizer
- Circular polarizer
- Elliptical polarizer

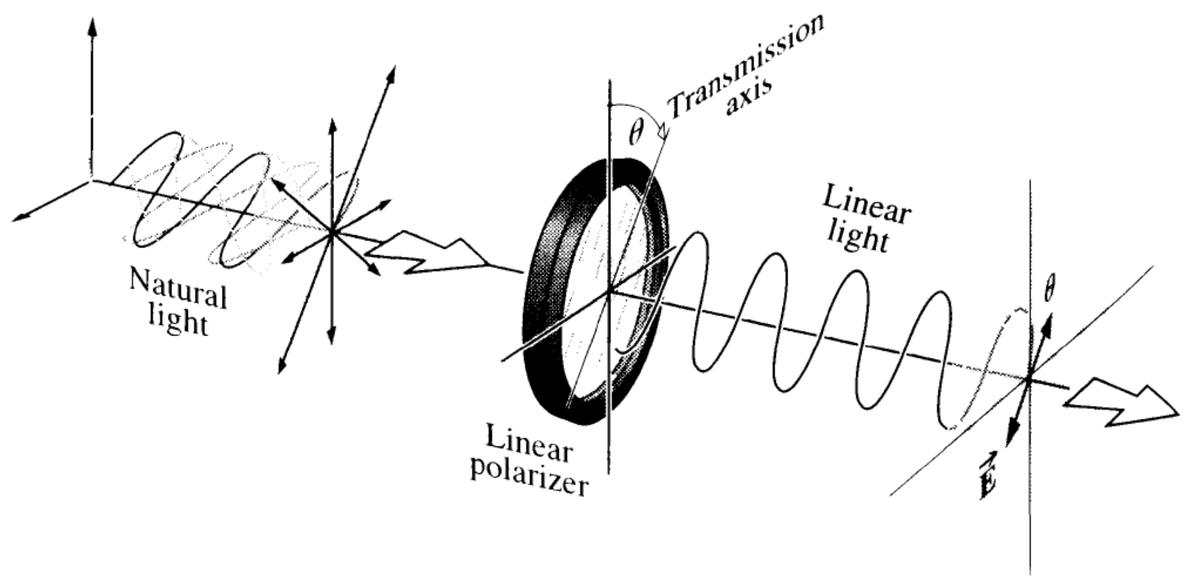
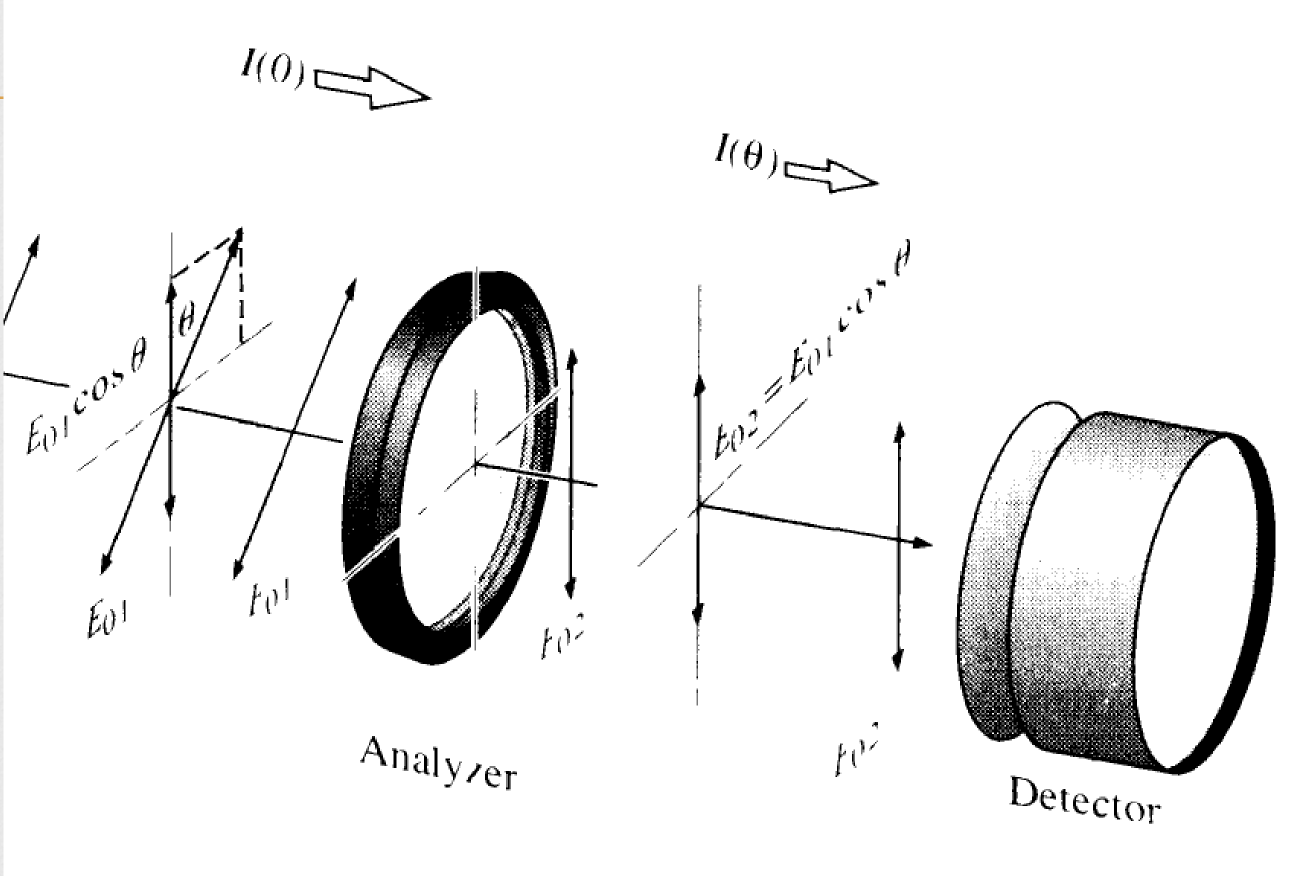


Figure 8.10 Natural light incident on a linear polarizer tilted at an angle θ with respect to the vertical.

Linear Polarizer



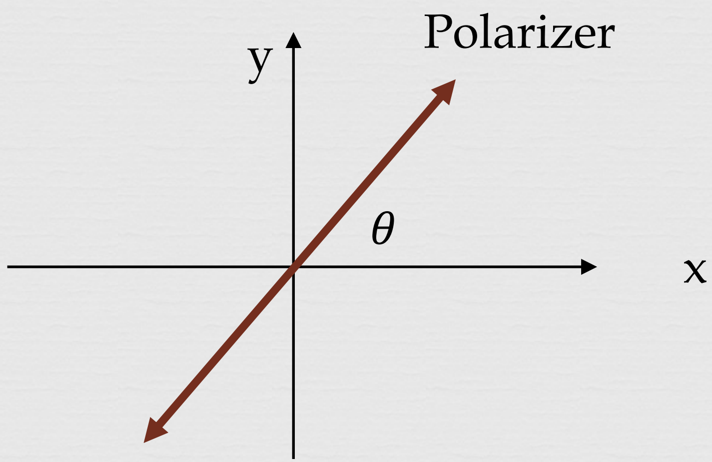
$$I(\theta) = \frac{c\epsilon_0}{2} E_{01}^2 \cos^2\theta$$

Question

input

$$E_x = E_{0x} \cos(kz - \omega t)$$

$$E_y = E_{0y} \cos(kz - \omega t + \varepsilon)$$



What is the output?

Retarders (Wave Plates)

$$\vec{\mathbf{E}}_x(z,t) = \hat{\mathbf{i}}\,E_{0x}\cos(kz - \omega t) = \hat{\imath}E_{0x}\cos(\omega t - kz) \Rightarrow \hat{\imath}E_{0x}\cos(\omega t)$$

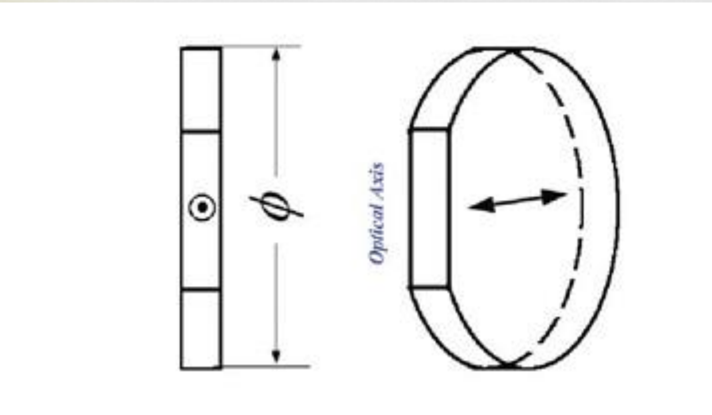
$$\vec{\mathbf{E}}_y(z,t) = \hat{\mathbf{j}}\,E_{0y}\cos(kz - \omega t + \varepsilon) = \hat{\jmath}E_{0y}\cos(\omega t - kz - \varepsilon) \Rightarrow \hat{\imath}E_{0y}\cos(\omega t - \varepsilon)$$

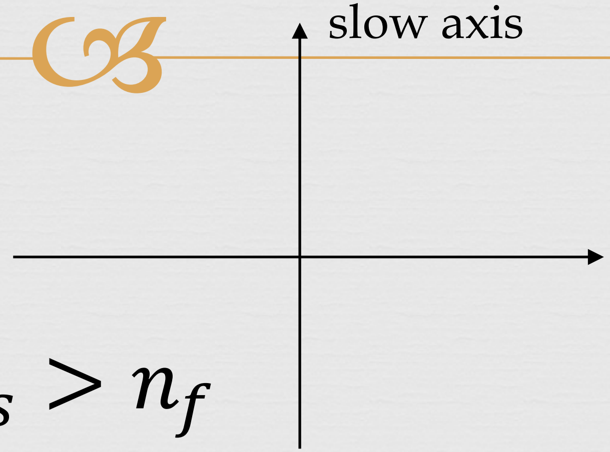
$$\vec{E}_x = \hat{\imath}E_{0x}\cos(\omega t - k_x z) \Rightarrow \hat{\imath}E_{0x}\cos(\omega t)$$

$$\vec{E}_y = \hat{\jmath}E_{0y}\cos(\omega t - k_y z - \varepsilon) \Rightarrow \hat{\jmath}E_{0y}\cos[\omega t - (k_y - k_x)z - \varepsilon]$$

$$\Delta\varphi = (k_y - k_x)z + \varepsilon$$

Retarders (Wave Plates)





$$k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda_0}$$

$$n_s > n_f$$

$$k_s = n_s k_0 > k_f = n_f k_0$$

$$\Delta\varphi_{fs} = (k_s - k_f)d = \frac{2\pi}{\lambda_0}\mathrm{d}(n_s - n_f)$$

$$\Delta\varphi_{fs} = \frac{1}{\#} \times 2\pi$$

1/# Wave plate

Half-Wave Plate

$$\Delta\varphi_{fs} = \frac{1}{2} \times 2\pi + 2m\,\pi = \pi + 2m\,\pi$$

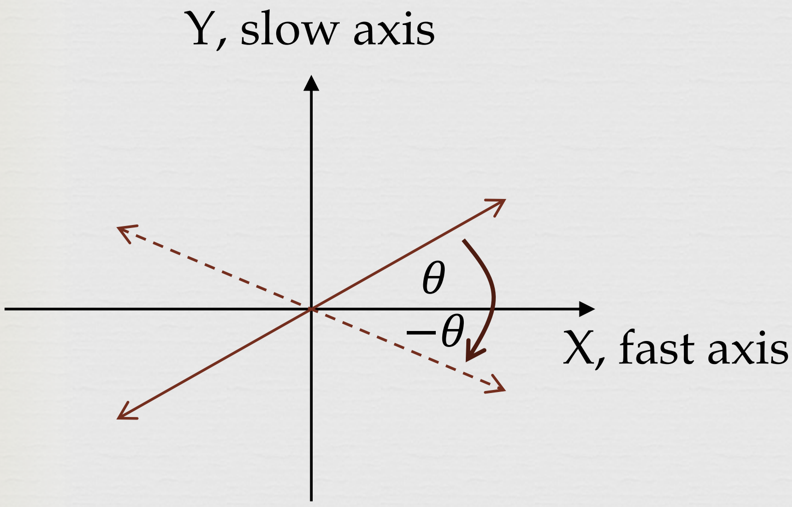
m=0, zero-order

m≠0, multiple-order

Input

$$\vec{E}_x = \hat{\imath} E_0 \cos\theta \cos(\omega t)$$

$$\vec{E}_y = \hat{\jmath} E_0 \sin\theta \cos(\omega t)$$

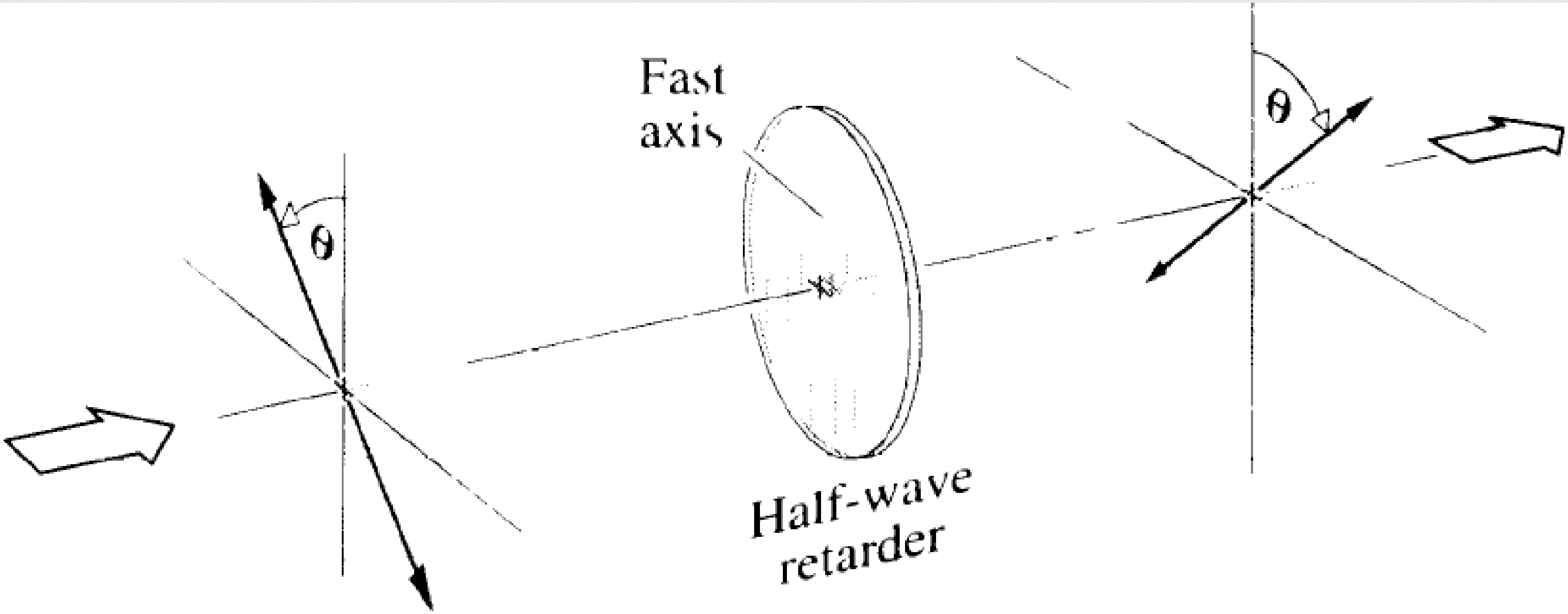


Output

$$\vec{E}_x = \hat{\imath} E_0 \cos\theta \cos(\omega t)$$
$$= \hat{\imath} E_0 \cos(-\theta) \cos(\omega t)$$
$$\vec{E}_y = \hat{\jmath} E_0 \sin\theta \cos(\omega t - \pi\ )$$
$$= -\hat{\jmath} E_0 \sin\theta \cos(\omega t\ )$$
$$= \hat{\jmath} E_0 \sin(-\theta) \cos(\omega t\ )$$

“Reflection” according to the fast axis

Half-Wave Plate



Quarter-Wave Plate

$\Delta\varphi_{fs} = \frac{1}{4} \times 2\pi + 2m\,\pi = \frac{\pi}{2} + 2m\,\pi$ m-order

m=0, zero-order

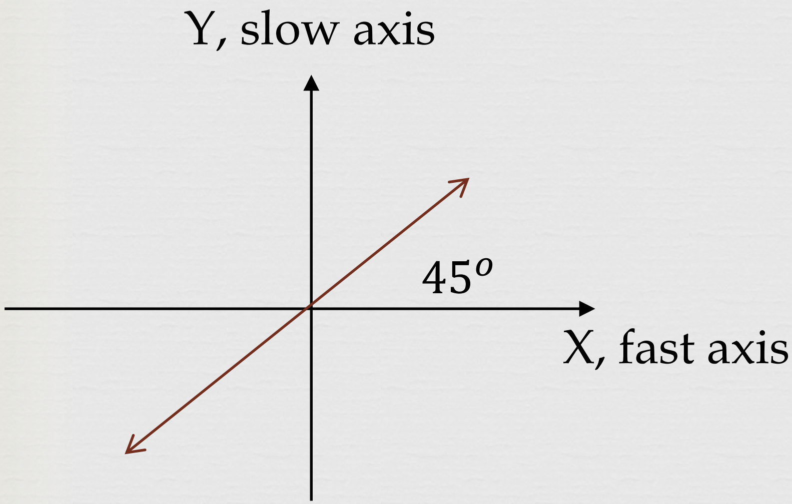
m≠0, multiple-order

Input

$$\vec{E}_x = \hat{\imath}\frac{1}{\sqrt{2}}E_0\cos(\omega t)$$

$$\vec{E}_y = \hat{\jmath}\frac{1}{\sqrt{2}}E_0\sin\theta\cos(\omega t)$$

Linearly polarized



Output

$$\vec{E}_x = \hat{\imath}\frac{1}{\sqrt{2}}E_0\cos(\omega t)$$

$$\vec{E}_y = \hat{\jmath}\frac{1}{\sqrt{2}}E_0\cos(\omega t - \pi/2)$$

$$=\hat{\jmath}\frac{1}{\sqrt{2}}E_0\sin(\omega t)$$

Left circularly polarized

Linear polarization ➔ Circular polarization

Quarter-Wave Plate

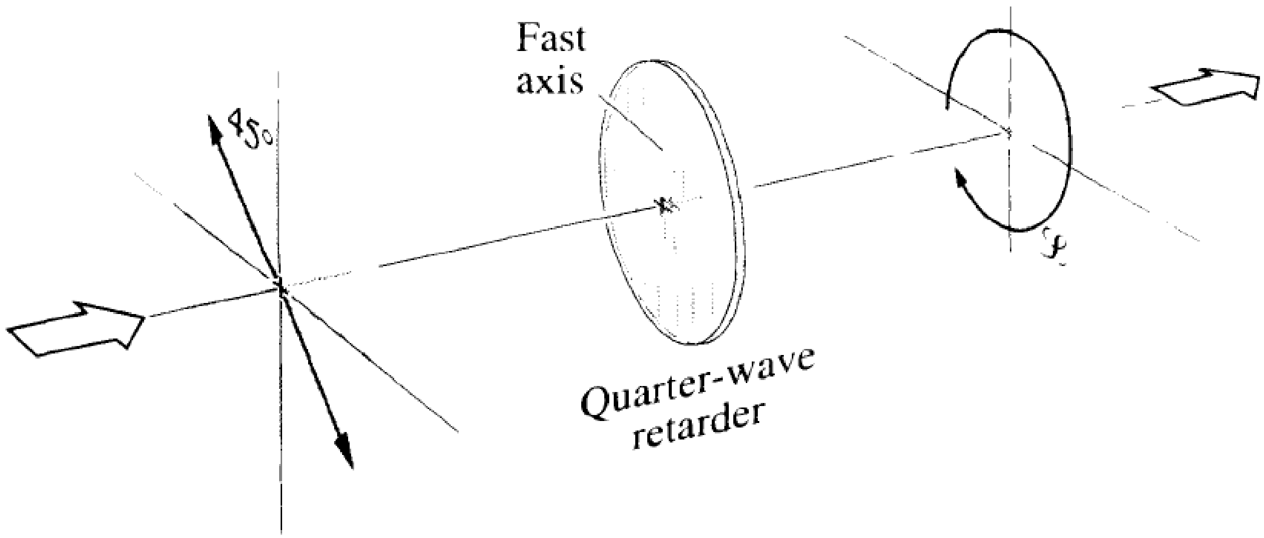


Figure 8.40 A quarter-wave plate transforms light initially linearly polarized at an angle 45° (oscillating in the first and third quadrants) into left circular light (rotating counterclockwise looking toward the source).

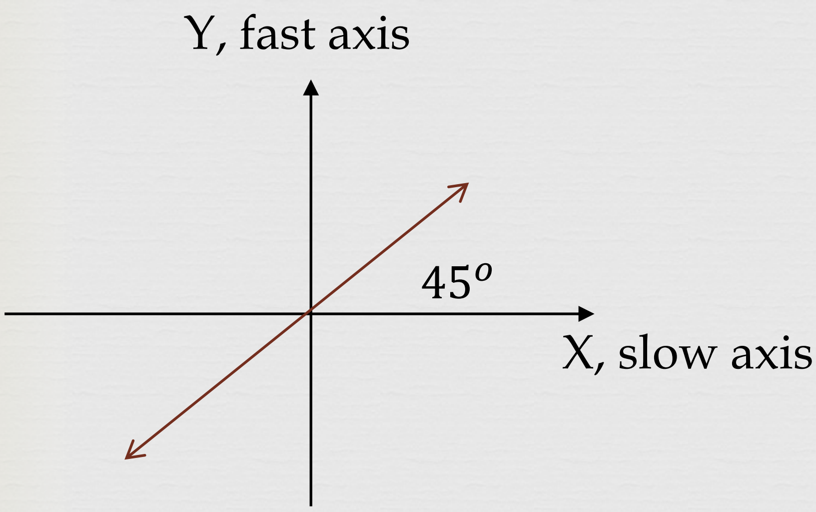
Quarter-Wave Plate

$$\Delta\varphi_{fs} = \frac{1}{4} \times 2\pi = \frac{\pi}{2}$$

Input $\quad \vec{E}_x = \hat{\imath}\frac{1}{\sqrt{2}}E_0\cos(\omega t)$

$\vec{E}_y = \hat{\jmath}\frac{1}{\sqrt{2}}E_0 \sin\theta \cos(\omega t)$

Linearly polarized



Output

$$\vec{E}_x = \hat{\imath}\frac{1}{\sqrt{2}}E_0 \cos(\omega t - \pi/2\ )$$

$$=\hat{\imath}\frac{1}{\sqrt{2}}E_0 \sin(\omega t\ )$$

$$\vec{E}_x = \hat{\jmath}\frac{1}{\sqrt{2}}E_0 \cos(\omega t)$$

Right circularly polarized

Linear polarization ➔ Circular polarization

Manipulating Polarization

- Combining a half-wave plate and quarter-wave plate, one can change a polarization to an arbitrary polarization.
- Combining a half-wave plate, quarter-wave plate, and a linear polarizer, one can construct an arbitrary polarizer.