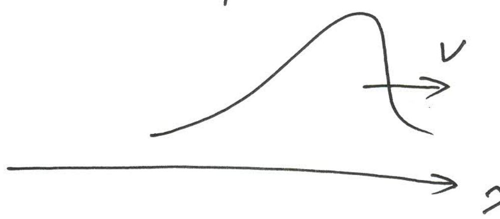
PHYS 3038 Optis ①



Ch2. Wave Motion

2.1 1D Waves

$$\psi(x,t) = f(x \mp vt) \begin{cases} - , & \text{move along } +x \text{ direction} \\ + , & \text{move along } -x \text{ direction} \end{cases}$$

2.1.1 1D Differential Wave Equation

$$x' = x \mp vt \quad \Rightarrow \quad \frac{\partial x'}{\partial x} = 1 \,, \quad \frac{\partial x'}{\partial t} = \mp v$$

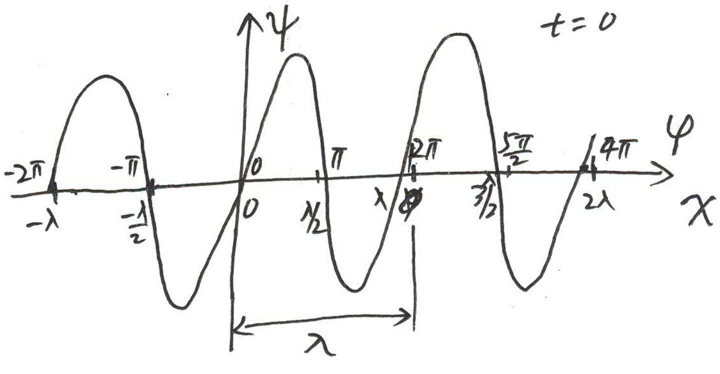
$$\left.\begin{aligned} \frac{\partial \psi}{\partial t} &= \frac{\partial f(x')}{\partial t} = \frac{\partial f(x')}{\partial x'}\frac{\partial x'}{\partial t} = \mp v \frac{\partial f}{\partial x'} \\ \frac{\partial \psi}{\partial x} &= \frac{\partial f(x')}{\partial x} = \frac{\partial f(x')}{\partial x'}\frac{\partial x'}{\partial x} = \frac{\partial f}{\partial x} \end{aligned}\right\} \Rightarrow \boxed{\frac{\partial \psi}{\partial t} = \mp v \frac{\partial \psi}{\partial x}}$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial t^2} = \mp v \frac{\partial^2 \psi}{\partial t \partial x} = \mp v \frac{\partial^2 \psi}{\partial x \partial t} = \mp v \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial t}\right) = v^2 \frac{\partial^2 \psi}{\partial x^2}$$

$$\boxed{\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2}} \iff \boxed{\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}} \qquad (2.11)$$

2.2 Harmonic Waves

$$\psi(x,t) = A \sin k(x - vt)$$



phase: $\varphi = k(x - vt)$

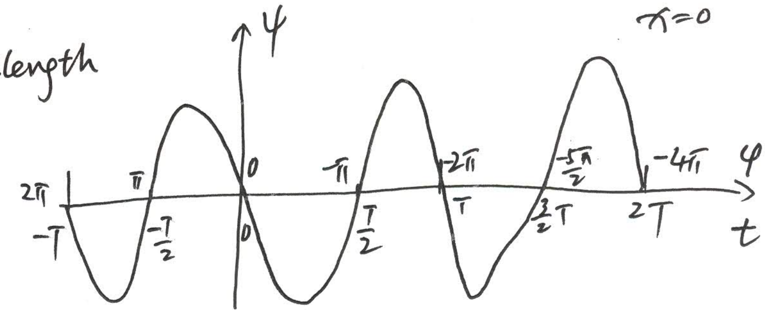
$$2\pi = k\lambda \Rightarrow \boxed{k = \frac{2\pi}{\lambda}}$$

propagation number (k) wavelength (λ)

$$2\pi = k \cdot (-v) \cdot (-T) = kvT$$

$$\Rightarrow \boxed{T = \frac{2\pi}{kv} = \frac{\lambda}{v}}$$ period

frequency $\boxed{\nu = \frac{1}{T} = \frac{v}{\lambda}}$



v : velocity

ν : frequency.

Angular frequency: $\boxed{\omega = \frac{2\pi}{T} = 2\pi\nu = \frac{2\pi}{\lambda}\lambda\nu = \frac{2\pi}{\lambda}\frac{\lambda}{T} = kv}$

2.3. phase and phase velocity

Recall: $\psi(x,t) = A\sin k(x-vt) = A\sin(kx-\omega t)$

phase $\varphi = kx - \omega t$

For a more general case $\psi(x,t) = A\sin(kx-\omega t+\varphi_0)$

phase $\varphi = kx - \omega t + \varphi_0$

φ_0 → initial phase at $x=0,\ t=0$.

For a constant phase $\varphi = kx - \omega t + \varphi_0 = \text{const}$, the position x is moving:

$$kx = \omega t - \varphi_0 + \text{const}$$

$$x = \frac{\omega}{k}t - \frac{\varphi_0}{k} + \frac{\text{const}}{k}$$

phase velocity ⇒ $\frac{dx}{dt} = \frac{\omega}{k} = v = \left(\frac{\partial x}{\partial t}\right)_\varphi$ (2.33)

$\boxed{\text{Youtube : Sine Wave in action}}$

2.4. The Superposition Principle

Recall: $\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$

If we have 2 solutions ψ_1 and ψ_2

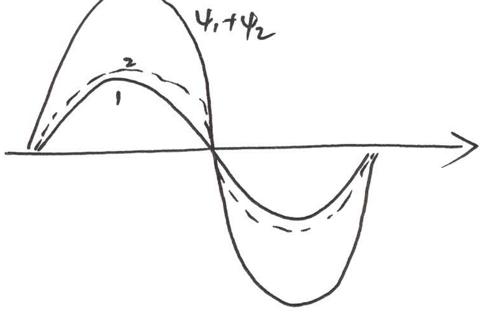
$$\left.\begin{aligned}\frac{\partial^2\psi_1}{\partial x^2} &= \frac{1}{v^2}\frac{\partial^2\psi_1}{\partial t^2}\\ \frac{\partial^2\psi_2}{\partial x^2} &= \frac{1}{v^2}\frac{\partial^2\psi_2}{\partial t^2}\end{aligned}\right\} \Rightarrow \frac{\partial^2(\psi_1+\psi_2)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2(\psi_1+\psi_2)}{\partial t^2}$$

⇒ $\psi_1+\psi_2$ is also a solution.

In general $a_1\psi_1 + a_2\psi_2$ is also a solution.

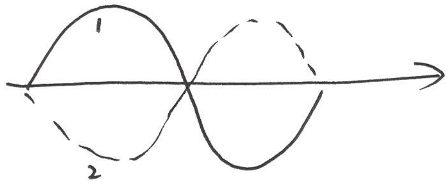
⇒ If we have N independent solutions $\psi_1, \psi_2, \dots, \psi_N$

a general solution $\boxed{\psi = \sum_n a_n\psi_n}$



In phase $\Rightarrow$ Constructive

$\varphi_1 = \varphi_2$



out of phase $\Rightarrow$ destructive

$\varphi_1 = -\varphi_2$

2.5 The Complex Wave

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad \boxed{i^2 = -1}$$

$$Re\{e^{i\theta}\} = \cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$

$$Im\{e^{i\theta}\} = \sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$

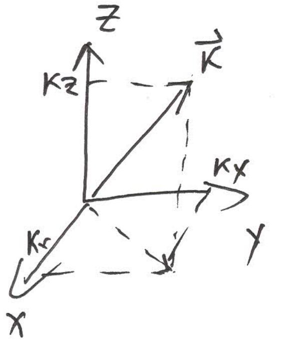
$$A e^{i(kx-\omega t+\varphi_0)} = A\cos(kx-\omega t+\varphi_0) + Ai\sin(kx-\omega t+\varphi_0)$$

Complex wave / harmonic $\quad \psi(x,t) = A e^{i(kx-\omega t+\varphi_0)}$

A Real wave $\quad A\cos(kx-\omega t+\varphi_0) = Re\{\psi(x,t)\}$

$$= \frac{1}{2}\left[\psi(x,t) + \psi^*(x,t)\right]$$

2.7 Plane Wave (3D)



$$A\cos(kx-\omega t) \longrightarrow A\cos(k_x x + k_y y + k_z z - \omega t)$$

$$= A\cos(\vec{k}\cdot\vec{r} - \omega t)$$

$$A e^{i(kx-\omega t)} \longrightarrow A e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z} = (k_x, k_y, k_z)$$

propagation vector

$$|\vec{k}| = k = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$k = \frac{2\pi}{\lambda} \iff \lambda = \frac{2\pi}{k}$$

$$v = \frac{\omega}{k}$$ phase velocity.

2.8. 3D differential Wave Equation. $\psi(x, y, z, t)$

1D: $$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\Downarrow$$

3D: $$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

or: $$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\Rightarrow \quad \boxed{\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}}$$

Solution:

$$\psi(x, y, z, t) = \psi(\vec{r}, t) = \cancel{f[\vec{k} \cdot \vec{r} - \vec{v}t}$$

$$= f\left(\frac{\vec{k} \cdot \vec{r}}{k} - vt\right)$$

Superposition principle $\boxed{\psi = \sum_n a_n \psi_n}$

2.9. Spherical Waves
2.10 Cylindrical Waves $\}$ self reading.