Name: _______________ Student ID:___________ Session: T ___

PHYS 3033 - Electricity and Magnetism I

Quiz 9

Time allowed: 20 minutes

10 Nov 2015

1. Check that $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}')}{\mathscr{r}} d\tau'$ is consistent with $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}')\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} d\tau'$.

2. Please prove that across a point on a surface at which the surface current density is **K**, the vector potentials above and below satisfy $\mathbf{A}_{\text{above}} = \mathbf{A}_{\text{below}}$. Consider both the parallel and perpendicular components of **A**!

Some of the formulae below may be useful:

$\nabla\cdot\mathbf{A} = 0$, $\nabla\times\mathbf{A} = \mathbf{B}$

$$\nabla\left(\frac{1}{\mathscr{r}}\right) = -\frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}$$

$$\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$$

$\oint \mathbf{B}d\mathbf{l} = \mu_0 I_{enc}$, $\mathbf{I} = \mathbf{K}/l_{perpendicular}$

Solution 1

1.

$$\nabla\times\mathbf{A}=\frac{\mu_0}{4\pi}\int\nabla\times\left(\frac{\mathbf{J}}{\mathrm{r}}\right)d\tau'=\frac{\mu_0}{4\pi}\int\left[\frac{1}{\mathrm{r}}(\nabla\times\mathbf{J})-\mathbf{J}\times\nabla\left(\frac{1}{\mathrm{r}}\right)\right]d\tau'.$$

But $\nabla\times\mathbf{J}=0$ (since $\mathbf{J}$ is not a function of $\mathbf{r}$), and $\nabla\left(\frac{1}{\mathrm{r}}\right)=-\frac{\hat{\mathbf{r}}}{\mathrm{r}^2}$, so

$$\nabla\times\mathbf{A}=\frac{\mu_0}{4\pi}\int\frac{\mathbf{J}\times\hat{\mathbf{r}}}{\mathrm{r}^2}d\tau'=\mathbf{B}.$$

2.

$$\nabla\cdot\mathbf{A}=0\Rightarrow\int(\nabla\cdot\mathbf{A})d\tau=\oint\mathbf{A}\cdot d\mathbf{a}=A^{\perp}_{\text{above}}-A^{\perp}_{\text{below}}=0\Rightarrow A^{\perp}_{\text{above}}=A^{\perp}_{\text{below}}.$$

$$\nabla\times\mathbf{A}=\mathbf{B}\Rightarrow\int(\nabla\times\mathbf{A})\cdot d\mathbf{a}=\oint\mathbf{A}\cdot d\mathbf{l}=\int\mathbf{B}\cdot d\mathbf{a}=0\Rightarrow\left(\mathbf{A}^{\parallel}_{\text{above}}-\mathbf{A}^{\parallel}_{\text{below}}\right)\cdot d\mathbf{l}=0\Rightarrow\mathbf{A}^{\parallel}_{\text{above}}=\mathbf{A}^{\parallel}_{\text{below}}.$$

So $\mathbf{A}_{\text{above}}=\mathbf{A}_{\text{below}}$.