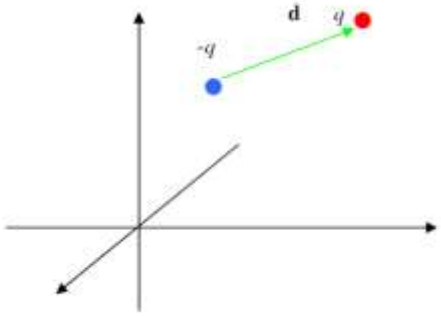
Name: _______________ Student ID: __________ Session: T ___

PHYS 3033 - Electricity and Magnetism I

Quiz 6: 13 Oct 2015

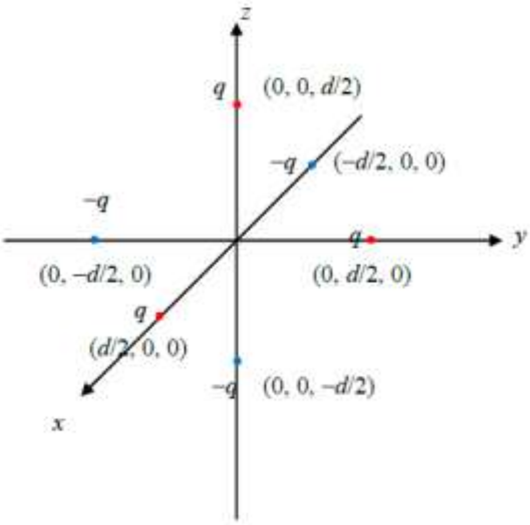
Time allowed: 15 minutes

(a) Show that the dipole moment of a pair of charges, $+q$ and $-q$, separated by a distance d, is coordinate-independent and given by $\mathbf{p} = q\mathbf{d}$, where $\mathbf{d}$ is a vector of magnitude d, pointing from $-q$ to q.



(b) In the following, we suppose that the dipole is oriented along the positive z direction. Find the electric field on a point on the z-axis at distance r ($r \gg d$) from the center of the dipole.

(c) Six charges are distributed as shown in the figure below. Find the monopole moment and dipole moment.



Useful formula:

Potential of an electric dipole:

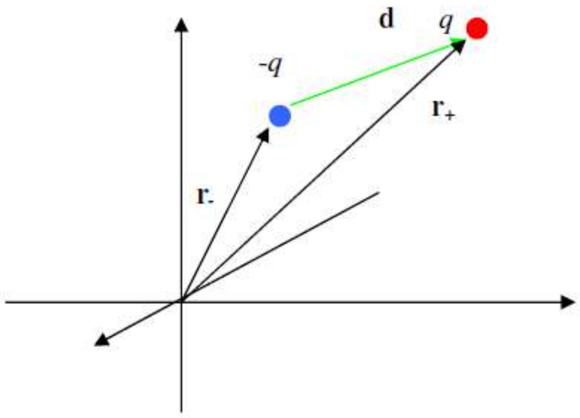
$$V_{\text{dip}}(\mathbf{r}) = \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2}$$

Gradient in spherical coordinates:

$$\nabla T = \frac{\partial T}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\boldsymbol{\varphi}$$

Solution

(a)



The dipole moment is defined by

$\mathbf{p} = \int \rho(\mathbf{r}')\mathbf{r}' d\tau'$ for continuous distribution and

$\mathbf{p} = \sum_i q_i \mathbf{r}_i$ for discrete distribution.

In this case, $\mathbf{p} = q\mathbf{r}_+ - q\mathbf{r}_- = q(\mathbf{r}_+ - \mathbf{r}_-) = q\mathbf{d}$, which is independent of the choice of coordinate system.

(b)

$V_{\text{dip}}(\mathbf{r}) = \dfrac{\mathbf{p}\cdot\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2}$ with **p** along *z* axis: $V_{\text{dip}}(\mathbf{r}) = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

$$\mathbf{E}_{\text{dip}} = -\nabla V_{\text{dip}}(\mathbf{r}) = -\hat{\mathbf{r}}\frac{\partial V_{\text{dip}}}{\partial r} - \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial V_{\text{dip}}}{\partial \theta} - \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial V_{\text{dip}}}{\partial \phi}$$

$$\mathbf{E}(r,\theta) = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}}\right)$$

(c)

Monopole moment $Q = q + q + q + (-q) + (-q) + (-q) = 0$.

Dipole moment

$$\mathbf{p} = q\frac{d}{2}\hat{\mathbf{x}} + (-q)\left(-\frac{d}{2}\hat{\mathbf{x}}\right) + q\frac{d}{2}\hat{\mathbf{y}} + (-q)\left(-\frac{d}{2}\hat{\mathbf{y}}\right) + q\frac{d}{2}\hat{\mathbf{z}} + (-q)\left(-\frac{d}{2}\hat{\mathbf{z}}\right)$$

$$= qd\hat{\mathbf{x}} + qd\hat{\mathbf{y}} + qd\hat{\mathbf{z}}$$

$$= qd\left(\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}\right)$$