Name: ________________ Student ID: ___________ Session: T ___

PHYS 3033 Electricity and Magnetism I

Quiz 5

6 October 2015

Time allowed 20 minutes

The surface of a hollow empty sphere is held at a constant potential $V_0(R,\theta) = V_0(\theta)$. From the method of separation of variables find the potential **inside** the sphere.

To solve this problem you may follow the proposed procedure below:

The general solution of the Laplace equation in spherical coordinates is:

$$\boxed{V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)}$$

1. Simplify this solution so that the radial component of the solution meets the boundary conditions at $r = R$.
2. Set the solution above at $r = R$ as equal $V_0(\theta)$. Multiply this expression by $P_m(\cos\theta)\sin\theta$ and then integrate from 0 to π.
3. Then take advantage of the fact that the solutions are Legendre polynomials $\mathrm{P}_l(\cos\theta)$ which are orthogonal functions in the interval from 0 to π in order to obtain an expression for the coefficients A_m and B_m. Here it may be useful to know that:
   $$\int_0^{\pi} P_l(\cos\theta)P_m(\cos\theta)\sin\theta d\theta = \frac{2}{2m+1}\delta_{lm}$$
4. As $V_0(\theta)$ is constant you may take it out of the integral as a prefactor and determine the coefficient A_m and B_m. For this it may be useful to expand the integral with the coefficient $P_0(\cos\theta) = 1$.
5. With these coefficients A_m and B_m write down the solution for the potential $V_0(r,\theta)$ **inside** the spherical volume.

See lecture notes for the solutions!