Name: _____ Student ID: _____ Session: T ___

PHYS 3033 Electricity and Magnetism I

Quiz 2

15 September 2015

Time allowed 15 minutes

Suppose the electric field in some region is found to be $\mathbf{E} = (kr^3 + hr^5)\hat{\mathbf{r}}$, in spherical coordinates (*k* and *h* are some constants).

(a) Find the charge density ρ.

(b) Find the total charge contained in a sphere of radius *R*, centered at the origin, by

(i) integrating the charge density in (a), and

(ii) using Gauss's law.

Solution

(a) $$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \rho = \varepsilon_0 \nabla \cdot \mathbf{E} = \varepsilon_0 \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \cdot \left(kr^3 + hr^5\right)\right) = 5\varepsilon_0 kr^2 + 7\varepsilon_0 hr^4$$

(b) (i) $$Q_{tot} = \int_0^R \int_0^\pi \int_0^{2\pi} \rho(r) r^2 \sin\theta \, dr d\theta d\phi = 5\varepsilon_0 k(4\pi) \int_0^R r^4 dr + 7\varepsilon_0 h(4\pi) \int_0^R r^6 dr$$
$$= 4\pi\varepsilon_0 kR^5 + 4\pi\varepsilon_0 hR^7$$

(ii) By Gauss's Law: $$Q_{tot} = \varepsilon_0 \oiint_{r=R} \mathbf{E} \cdot d\mathbf{a} = \varepsilon_0 \left(kR^3\right)\left(4\pi R^2\right) = 4\pi\varepsilon_0 kR^5 + 4\pi\varepsilon_0 hR^7$$