

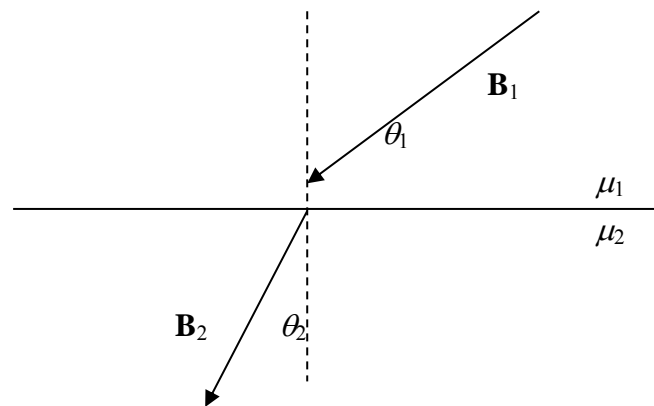
PHYS 3033 - Electricity and Magnetism I

Quiz 9

Time allowed: 15 minutes

20 Nov 2015

Just above and below the interface between two linear magnetic media the directions of the magnetic fields change, as shown in the figure below.



Show that $\tan \theta_2 / \tan \theta_1 = \mu_2 / \mu_1$, assuming there is no free current at the boundary.

Solutions:

It is always true that $\nabla \cdot \mathbf{B} = 0$, and if there is no free current, $\nabla \times \mathbf{H} = \mathbf{0}$. The two boundary conditions are

$$\begin{aligned} & \begin{cases} \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{0} \end{cases} \Rightarrow \begin{cases} B_{2\perp} - B_{1\perp} = 0 \\ H_{2\perp} - H_{1\perp} = 0 \end{cases} \Rightarrow \begin{cases} B_{1\perp} = B_{2\perp} \\ H_{1\perp} = H_{2\perp} \end{cases} \\ \Rightarrow & \begin{cases} B_{1\perp} = B_{2\perp} \\ B_{1\perp} / \mu_1 = B_{2\perp} / \mu_2 \end{cases} \Rightarrow \begin{cases} B_1 \cos \theta_1 = B_2 \cos \theta_2 & \text{----- (1)} \\ B_1 \sin \theta_1 / \mu_1 = B_2 \sin \theta_2 / \mu_2 & \text{----- (2)} \end{cases} \end{aligned}$$

Divide (2) by (1), hence

$$\frac{B_1 \sin \theta_1 / \mu_1}{B_1 \cos \theta_1} = \frac{B_2 \sin \theta_2 / \mu_2}{B_2 \cos \theta_2} \Rightarrow \frac{\tan \theta_2}{\tan \theta_1} = \frac{\mu_2}{\mu_1}$$