Name: _______________ Student ID: ___________ Session: T ___

PHYS 3033 - Electricity and Magnetism I

Quiz 1

Time allowed: 15 minutes

8 Sep 2015

1. (a) Use Stokes' theorem to show that $\nabla \times (\nabla T) = \mathbf{0}$ for any scalar field *T*.

   (b) Use divergence theorem to show that $\nabla \cdot (\nabla \times \mathbf{A}) = 0$ for any vector field **A**.

2. Consider the vector field $\mathbf{F} = \frac{1}{r^2}\hat{\mathbf{r}}$. Compute the line integral $\int_C -\mathbf{F} \cdot d\mathbf{l}$, where *C* is the straight path starting at (r_0, θ, ϕ) and ending at (r, θ, ϕ) (in spherical coordinates). What is the line integral in the limit when $r_0 \to \infty$?

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Quiz 1 Solution

1. (a) By Stokes' theorem, for any *T* and for arbitrary surface *S*,

$$\int_S \left(\nabla\times(\nabla T)\right)\cdot d\mathbf{a} = \oint_C \nabla T\cdot d\mathbf{l}\,,$$

where *C* is the boundary of *S* (and is therefore a closed path).
By the fundamental theorem of calculus,

$$\oint_C \nabla T\cdot d\mathbf{l} = 0\,,$$

for any closed path *C*.

Therefore $\int_S \left(\nabla\times(\nabla T)\right)\cdot d\mathbf{a} = 0$ for any surface *S*, and hence

$$\nabla\times(\nabla T) = \mathbf{0}\,.$$

(b) By divergence theorem, for any **A** and for arbitrary region ς,

$$\int_V \nabla\cdot(\nabla\times\mathbf{A})\,d\tau = \oint_S (\nabla\times\mathbf{A})\cdot d\mathbf{a}\,,$$

where *S* is the boundary of ς (and is therefore a closed surface).
By Stokes' theorem $\int_S (\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint_{\text{Boundary of } S} \mathbf{A}\cdot d\mathbf{l}\,.$

For a closed surface, there is no boundary.

So $\oint_S (\nabla\times\mathbf{A})\cdot d\mathbf{a} = 0$ for arbitrary closed surface *S*.

This implies $\int_V \nabla\cdot(\nabla\times\mathbf{A})\,d\tau = 0$ for arbitrary region ς, and hence

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0\,.$$

2.

$$\begin{aligned} -\int \mathbf{F}\cdot d\mathbf{l} &= -\int_{r_0}^{r} \frac{1}{r^2}\hat{\mathbf{r}}\cdot dr\hat{\mathbf{r}} \\ &= -\int_{r_0}^{r} \frac{1}{r^2}\,dr \\ &= \frac{1}{r} - \frac{1}{r_0} \end{aligned}$$

When $r_0 \to \infty$, $-\int_C \mathbf{F}\cdot d\mathbf{l} = \frac{1}{r}$