PC2174

Tutorial 6: Higher Order Ordinary Differential Equations

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1. Find the general solutions of

   (a) $\frac{d^3y}{dx^3} - 12\frac{dy}{dx} + 16y = 32x - 8$

   (Hint: Roots of auxiliary equation are integers and lie between -5 and 5.)

   (b) $\frac{d}{dx}\left(\frac{1}{y}\frac{dy}{dx}\right) + (2a\coth 2ax)\left(\frac{1}{y}\frac{dy}{dx}\right) = 2a^2$

   where a is a constant. (Hint: $\int \operatorname{cosech} 2ax\, dx = (2a)^{-1}\ln(|\tanh ax|)$.)

2. The quantities $x(t)$, $y(t)$ satisfy the simultaneous equations

$$\begin{aligned} \ddot{x} + 2n\dot{x} + n^2x &= 0 \\ \ddot{y} + 2n\dot{y} + n^2y &= \mu\dot{x}, \end{aligned}$$

   where $x(0) = y(0) = \dot{y}(0) = 0$ and $\dot{x}(0) = \lambda$. Show that

$$y(t) = \frac{1}{2}\mu\lambda t^2\left(1 - \frac{1}{3}nt\right)\exp(-nt).$$

3. Find the general solution of

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = x,$$

   given that $y(1) = 1$ and $y(e) = 2e$.

4. Use the method of variation of parameters to find the general solutions of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2xe^x.$$