PC2174

Tutorial 5: First Order Ordinary Differential Equations

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1. A radioactive isotope decays in such a way that the number of atoms present at a given time, $N(t)$, obeys the equation
$$\frac{dN}{dt} = -\lambda N.$$
If there are initially N_0 atoms present, find $N(t)$ at later times.

2. By finding an appropriate integrating factor solve
$$\frac{dy}{dx} = -\frac{2x^2 + y^2 + x}{xy}.$$

3. Solve
$$(y - x)\frac{dy}{dx} + 2x + 3y = 0.$$

4. A mass m is accelerated by a time-varying force $\exp(-\beta t)v^3$, where v is its velocity. It also experiences a resistive force ηv, where η is a constant, owing to its motion through the air. The equation of motion of the mass is therefore given by
$$m\frac{dv}{dt} = \exp(-\beta t)v^3 - \eta v.$$
Find an expression for the velocity v of the mass as a function of time, given that it has an initial velocity v_0.

5. By treating y as the independent variable, show that the general solution of
$$(x + y^3)\frac{dy}{dx} = y$$
is $x = y(A + \frac{1}{2}y^2)$.