PC2174

Tutorial 4: Integral Transforms

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1. Find the Fourier transform of the function $f(t) = \exp(-|t|)$.

   (a) By applying Fourier's inversion theorem prove that

   $$\frac{\pi}{2}\exp(-|t|) = \int_0^\infty \frac{\cos\omega t}{1+\omega^2}d\omega.$$

   (b) By making the substitution $\omega = \tan\theta$, demonstrate the validity of Parseval's theorem for this function.

2. By taking the Fourier transform of the equation

   $$\frac{d^2\phi}{dx^2} - K^2\phi = f(x)$$

   show that its solution $\phi(x)$ can be written as

   $$\phi(x) = \frac{-1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{e^{ikx}\tilde{f}(k)}{k^2+K^2}dk,$$

   where $\tilde{f}(k)$ is the Fourier transform of $f(x)$.