PC2174

Tutorial 3: Fourier Series

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1. Prove the following orthogonality relations:

$$\int_{x_0}^{x_0+L} \sin\left(\frac{2\pi r x}{L}\right)\cos\left(\frac{2\pi p x}{L}\right) dx = 0$$

$$\int_{x_0}^{x_0+L} \cos\left(\frac{2\pi r x}{L}\right)\cos\left(\frac{2\pi p x}{L}\right) dx = \begin{cases} L, & r = p = 0 \\ \frac{1}{2}L, & r = p > 0 \\ 0, & r \neq p \end{cases}$$

$$\int_{x_0}^{x_0+L} \sin\left(\frac{2\pi r x}{L}\right)\sin\left(\frac{2\pi p x}{L}\right) dx = \begin{cases} 0, & r = p = 0 \\ \frac{1}{2}L, & r = p > 0 \\ 0, & r \neq p \end{cases}$$

2. Find the Fourier series of the functions $f(x) = x$ in the range $-\pi < x <\leq \pi$. Hence show that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}.$$

3. Find the Fourier coefficients in the expansion if $f(x) = \exp x$ over the range $-1 < x < 1$. What value will the expansion have when $x = 2$?

4. By integrating term by term the Fourier series found in the previous question, show that $\int \exp x \, dx = \exp x + c$. Why is it not possible to show that $d(\exp x)/dx = \exp x$ by differentiating the Fourier series of $f(x) = \exp x$ in a similar manner?

5. Express the function $f(x) = x^2$ as a Fourier sine series in the range $0 < x \leq 2$, and show that it converges to zero at $x = \pm 2$.