Formulae

Fourier Series

The Fourier series expansion of a function $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{r=1}^{\infty} \left[a_r \cos\left(\frac{2\pi r x}{L}\right) + b_r \sin\left(\frac{2\pi r x}{L}\right) \right]$$

where

$$\begin{aligned} a_r &= \frac{2}{L} \int_{x_0}^{x_0+L} f(x) \cos\left(\frac{2\pi r x}{L}\right) dx \\ b_r &= \frac{2}{L} \int_{x_0}^{x_0+L} f(x) \sin\left(\frac{2\pi r x}{L}\right) dx \end{aligned}$$

The complex Fourier series expansion is written as

$$f(x) = \sum_{r=-\infty}^{\infty} c_r \exp\left(\frac{2\pi i r x}{L}\right),$$

where

$$c_r = \frac{1}{L} \int_{x_0}^{x_0+L} f(x) \exp\left(-\frac{2\pi i r x}{L}\right) dx$$

Fourier Transform

Fourier transform of $f(t)$ is defined by

$$\tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt,$$

and its inverse by

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{f}(\omega) e^{i\omega t}\, d\omega.$$

Exact equation

An exact first-degree first-order ODE is of the form

$$A(x,y)\, dx + B(x,y)\, dy = 0$$

and $\frac{\partial A}{\partial y} = \frac{\partial B}{\partial x}$.

Bernoulli's equation

Bernoulli's equation has the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n \text{ where } n \neq 0 \text{ or } 1$$

This equation is solved by using the substitution $v = y^{1-n}$.