Physics 125b – Problem Set 11 – Due Feb 5, 2008

Version 1 – Jan 30, 2008

This problem set continues our study of Shankar Chapter 12 and Lecture Notes 13 on angular momentum in three dimensions and the full solutions to the Schrödinger Equation in spherical coordinates.

Many basic problems in QM can be found in textbooks – there are only so many solvable elementary problems out there. Please refrain from using solutions from other textbooks. Obviously, you will learn more and develop better intuition for QM by solving the problems yourself. We are happy to provide hints to get you through the tricky parts of a problem, but you *must* learn to set up and solve these problems from scratch by yourself.

1. Use whatever software you would like (Matlab, Mathematica, IDL, etc.) to plot contours of constant $|Y_l^m(\theta, \phi = 0)|^2$ in the xz-plane for $l = 0, 1, 2, 3$ and $m = 0, \cdots, l$. For example, your plot for $l = 0$ should be a circle centered on the origin. (You may look up the Y_l^m; you don't need to derive them!)

2. Shankar 12.5.4. Don't be misled, this problem should not require a lot of algebra.

3. Shankar 12.5.14. Remember that the Y_l^m are just the position-basis representation of the $|l, m\,\rangle$ eigenstates; use this to avoid Y_l^m algebra. Also, you may use the results of Shankar 12.5.5.

4. Shankar 12.6.9. It may seem like there is no guidance on finding the allowed form for the wavefunction for $r > r_0$; realize that you may reuse the derivation of spherical Bessel and Neumann functions with k replaced by $i\,\kappa$ and by using the spherical Hankel functions $h_l(\rho) = j_l(\rho) + i\,\eta_l(\rho)$ to obtain solutions with the desired characteristics for $r \to \infty$.

5. Shankar 12.6.10. Also, find the probability of obtaining a particular eigenvalue $l(l+1)\,\hbar^2$ of L^2 relative to the probability of obtaining the zero eigenvalue of L^2. What are the expectation values of L_x, L_y, and L_z? Hints:

   - In addition to Shankar's hint, ask yourself – can C_l depend on r?
   - You will happen on a difficult integral. You could calculate it using Shankar's equations (2) and (3), but you may just use the following result if you don't want to spend your time on it:
   $$\int_{-1}^{1} dx\, x^l P_l(x) = \frac{2^{l+1} l!\, l!}{(2\,l+1)!} \quad (1)$$
   - You do not need to do terrible integrals with spherical harmonics to find the expectation values requested above.