

Physics 125a – Problem Set 8 – Due Dec 7, 2007

Version 1 – Nov 30, 2007

Note the unusual due date!

This problem focuses on systems of identical particles, on coordinate and symmetry transformations, and on the Ehrenfest Theorem – Shankar Chapters 10, 11, and 6, and Lecture Notes 8, 9, and 10.

Many basic problems in QM can be found in textbooks – there are only so many solvable elementary problems out there. Please refrain from using solutions from other textbooks. Obviously, you will learn more and develop better intuition for QM by solving the problems yourself. We are happy to provide hints to get you through the tricky parts of a problem, but you *must* learn to set up and solve these problems from scratch by yourself.

1. Shankar 10.3.1
2. Shankar 10.3.4
3. Shankar 10.3.5. Feel free to use as much of our formalism for coordinate transformations as you like for doing this problem – the particle-exchange operator can be interpreted as a coordinate transformation.
4. Rotation symmetry transformation of the isotropic two-dimensional SHO. Recall from Problem Set 7 the two-dimensional SHO,

$$H = \frac{P_x^2 + P_y^2}{2m} + \frac{1}{2} m \omega^2 (X^2 + Y^2) \quad (1)$$

Consider a coordinate transformation involving rotation in the plane by an angle θ (CCW when viewed from above):

$$x' = x \cos \theta + y \sin \theta \quad y' = -x \sin \theta + y \cos \theta \quad (2)$$

Answer the following:

- (a) Whether regarded as passive or active, the transformation creates a new set of basis kets $\{|x', y'\rangle\}$ where x' and y' are relative to the new coordinate axes. Given a transformed basis ket $|x' = u, y' = v\rangle$, what element of the untransformed basis kets $\{|x, y\rangle\}$ is it equal to? That is, find a formula for x and y in $|x, y\rangle$ in terms of u and v . This will be easy if you have understood the class material.
- (b) There will be new position and momentum operators associated with the new basis, X', Y', P'_x, P'_y . Write these operators in terms of the untransformed operators $X, Y, P_x,$ and P_y . Writing $O' = T O T^\dagger$ is of course not sufficient. We are looking for something like the relation $X' = Y$ for the mirror transformation example from the lecture notes or $X' = -X$ for the parity transformation example.

- (c) Consider the first three states $(n_x, n_y) = (0, 0)$, $(1, 0)$, and $(0, 1)$ that we looked at in PS7 in terms of the untransformed basis (nothing tricky about that). Write the representation of the states in terms of the transformed basis $\{|x', y'\rangle\}$. Also, considering the transformation as active, write both the representation of the transformed versions of the above three states in terms of the transformed basis and of the transformed states in terms of the untransformed basis. Again, refer to the mirror transformation example in the lecture notes if this is unclear.
- (d) Show that the rotation transformation is a symmetry transformation of H . Why would it not be a symmetry transformation if the SHO were anisotropic? Check this in detail algebraically; “because it is anisotropic” is not a sufficient answer.
- (e) What is the conserved quantity associated with this symmetry transformation?
5. Derive the Euler-Lagrange equations for a one-dimensional system under the influence of a potential $V(x)$ via the Ehrenfest Theorem.