Physics 125b – Final Exam – Due Mar 21, 2008

Instructions

Material: All lectures this term, though emphasis will be on second half (Jan 28 onward), lecture notes 11 onward. This corresponds roughly to Shankar Chapters 12 through 17, but if material from Shankar was not covered by or pointed to in the lecture notes or homework, it will not be needed here (you *are* responsible for all of Ch. 13 on the hydrogen atom). Review the material ahead of time, consult me, the TAs, your fellow students, or other texts if there is material you are having trouble with.

Logistics: The exam consists of this page plus 2 pages of exam questions, a total of 5 questions. Do not look at the exam until you are ready to start it. Please use a blue book if possible (makes grading easier), but there will be no penalty if you don't have one.

Time: 4 hrs, fixed time. You may take as many breaks as you like, but they may add up to no more than 30 minutes (2 x 15 minutes, 3 x 10 minutes, etc.).

Reference policy: Shankar, official class lecture notes, problem sets and solutions, your own lecture notes or other notes you have taken to help yourself understand the material. No other textbooks, no web searches, no interaction with your fellow students. You may use a computer to write up your exam, but calculators and symbolic manipulation programs are neither needed or allowed. If you write up with a computer, it must be done within the 4-hour exam period; no additional time is allowed for transcription or proofreading. No dispensations will be given for technical difficulties. You may quote without proof any results given in the lecture notes, problem sets, or in Shankar.

Due date: Friday, Mar 21, 4 pm, my office (311 Downs). 4 pm means 4 pm. Late exams will require extenuating circumstances; otherwise, no credit will be given.

Grading: Each problem is 10 points for a total of 50 points. The exam is 1/3 of the class grade.

Suggestions on taking the exam:

- Go through and figure out roughly how to do each problem first; make sure you've got the concept straight before you start writing.
- Don't fixate on a particular problem. They are not all of equal difficulty. Come back to ones you are having difficulty with.
- Don't get buried in algebra. Get each problem to the point where you think you will get most of the points, then come back and worry about the algebra.

1. Consider an electron moving in a spherically symmetric potential $V = k\,r$, $k > 0$

   (a) Use the uncertainty principle to estimate the ground state energy.

   (b) Use the Bohr-Sommerfeld quantization rule to calculate the ground state energy. (Remember how we dealt with the wall on the midterm exam?)

   (c) Set up and describe how you would do a variational calculation of the ground state energy using the exponential trial wavefunction, $\psi(\vec{r}) = e^{-\lambda\,r}$. Do not go through the algebra of evaluating the integrals.

   (d) Write down the exact ground state wavefunction. Write down a condition that will give the true ground state energy (but don't try to solve for E_0). Hint: where do Airy functions in the WKB technique come from?

   (e) Write down the effective potential for nonzero angular momentum states.

2. Consider a hydrogen atom whose nucleus is at the origin, and let there be an infinite potential wall in the xy plane.

   (a) Find the explicit form of the ground state wavefunction(s).

   (b) Find all other eigenstates in terms of the standard R_{nl} and Y_l^m radial eigenfunctions and spherical harmonics applied for the hydrogen atom problem.

   Hint: recall from the midterm exam the effect of the wall in a one-dimensional problem, and recall from class the parity properties and ϕ-dependence of the spherical harmonics. To check that you found all the states, you may find it useful to consider the analogous problem with the wall in the xz plane and to recall $Y_l^{-m} = (-1)^m (Y_l^m)^*$ (Shankar 12.5.40). (Think about the ϕ-dependence of the spherical harmonics). We choose not to do the xz plane problem because L_z does not commute with the Hamiltonian for that problem.

3. Consider the unperturbed hydrogen atom, neglecting the electron spin and relativistic and spin-orbit corrections. Under these assumptions, the $n = 2$ level is known to be four-fold degenerate: the $|n, l, m\,\rangle$ states $|2, 0, 0\,\rangle$, $|2, 1, 1\,\rangle$, $|2, 1, 0\,\rangle$, and $|2, 1, -1\,\rangle$ all have the same energy. Consider the effect of a perturbing potential

   $$\delta H^{(1)} = f(r)\,x\,y \tag{1}$$

   where $f(r)$ is an unspecified but well-behaved function of radius only (does not diverge at the origin, drops off sufficiently quickly as $r \to \infty$). With this perturbation treated to first order, determine:

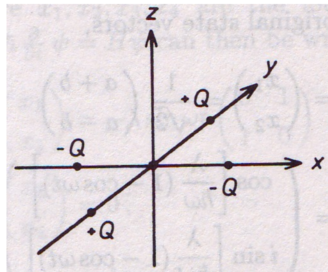
   (a) How many distinct energy levels are there?

   (b) What is the degeneracy of each level?

   (c) Given the energy shift A, $A > 0$, for one of the levels, what are the values of the shifts of the other energy levels?

   You will receive full credit for doing this problem using spherical tensor operators and the Wigner-Eckart theorem, and half credit if you do it by other means.

4. An ion of a certain atom has $l = 1$ and $s = 0$ when it is in free space. The ion is implanted in a crystalline solid (at $(x, y, z) = (0, 0, 0)$) and sees a local environment of 4 point charges as shown in the figure.



One can show by applying the Wigner-Eckart theorem (don't try this!) that the effective Hamiltonian caused by this environment is

$$H_1 = \frac{\alpha}{\hbar^2}\left(L_x^2 - L_y^2\right) \tag{2}$$

where α is a constant. In addition, a magnetic field is applied in the z direction and causes an additional Hamiltonian

$$H_2 = \frac{\beta}{\hbar}\, B\, L_z \tag{3}$$

You may assume $\alpha > 0$, $\beta > 0$.

(a) Express the Hamiltonian $H = H_1 + H_2$ in terms of the $L_\pm$ raising and lower operators.

(b) Find the matrix of the Hamiltonian in the basis set of the three states $|l, m\rangle = |1, 1\rangle$, $|1, 0\rangle$, and $|1, -1\rangle$.

(c) Find the energy levels of the ion as a function of B. Sketch the result.

(d) When $B = 0$, what are the eigenstates of the ion?

5. The ground state of the true helium atom is nondegenerate. Consider a hypothetical helium atom in which the two electrons are replaced by two identical spin-1 particles of negative unit electric charge. Neglect all perturbations. For this atom, what is the degeneracy of the ground state? (Explain your reasoning, of course.)