COMP3711: Design and Analysis of Algorithms

Tutorial 10

HKUST

Question 1

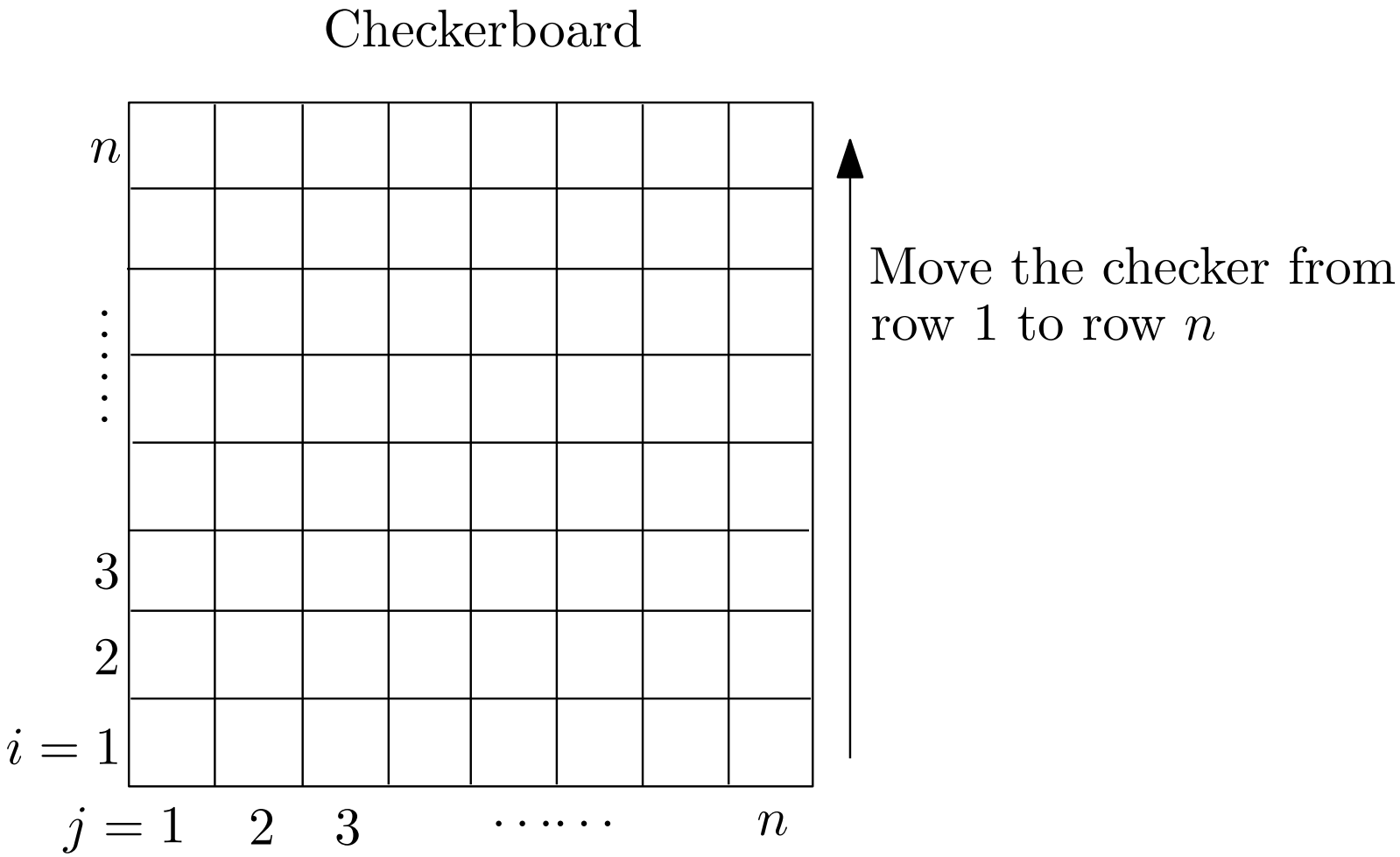
Let G be a connected undirected graph with distinct weights on the edges, and let e be an edge of G. Suppose e is the largest-weight edge in some cycle of G. Show that e cannot be in the MST of G.

Question 2

Suppose that you are given an $n \times n$ checkerboard and a checker. You must move the checker from the bottom edge of the board to the top edge of the board according to the following rule. At each step you may move the checker to one of three squares:

1) the square immediately above,
2) the square that is one up and one to the left (but only if the checker is not already in the leftmost column),
3) the square that is one up and one to the right (but only if the checker is not already in the rightmost column).

Question 2


Checkerboard
n
3
2
i = 1
j = 1 2 3 ..... n
Move the checker from row 1 to row n

Question 2

Each time you move from square (i, j) to square (i', j'), you receive $p((i, j), (i', j'))$ dollars. You are given a list of the values $p((i, j), (i', j'))$ for each pair $((i, j), (i', j'))$ for which a move from (i,j) to (i',j') is legal. Do not assume that $p((i, j), (i', j'))$ is positive.

Give an algorithm that figures out the set of moves that will move the checker from somewhere along the bottom edge to somewhere along the top edge while gathering as many dollars as possible. You algorithm is free to pick any square along the bottom edge as a starting point and any square along the top edge as a destination in order to maximize the number of dollars gathered along the way. What is the running time of your algorithm?

Question 3

Prim's minimum spanning tree algorithm and Dijkstra's shortest path algorithm are very similar, but with crucial differences. Run both algorithms on the following graph, and show the partial MST / shortest path tree after every new edge has been added. The starting vertex for both algorithms is "a".