COMP3711: Design and Analysis of Algorithms

Tutorial 8

HKUST

Question 1

A string of parentheses is said to be balanced if the left- and right-parentheses in the string can be paired off properly. For example, the strings (()) and ()() are both balanced, while the string (()))(is not. Given a string S of length n consisting of parentheses, design an algorithm to find the longest subsequence of S that is balanced.

Solution 1

Step1: Space of subproblem

For $1 \leq i \leq n, 1 \leq j \leq n$, define $D[i,j]$ be the longest balanced subsequence of the substring $S[i..j]$.

Step2: Recursive formulation

Base cases ($i \geq j$): $D[i,j] = 0$.

Recursive cases ($1 \leq i < j \leq n$):

$$D[i,j] = \max \begin{cases} [1]\, D[i+1, j-1] + 2 & S[i] =' (' \text{ and } S[j] =')' \\ [2]\, \max\limits_{i<k<j} \{D[i,k] + D[k+1,j]\} & \\ [3]\, D[i+1, j] & \\ [4]\, D[i, j-1] & \end{cases}$$

Step3: Bottom-up computation

Just like Matrix Multiplication Problem, we compute the $D[i, j]$ in increasing order of $|j - i|$.

Step4: Construction optimal solution

For $1 \leq i \leq n, 1 \leq j \leq n$, define $c[i, j]$ to record which case we choose to get the solution for the subproblem. Then we can start from $c[1, n]$ and recursively construct the optimal solution.

Question 2

Let $G = (V, E)$ be an undirected graph where V is the set of vertices and E is the set of edges.

a) What is the maximum number of edges in G?
b) What is the maximum number of edges in G if two vertices has degree 0.
c) What is the maximum number of edges in G if G is acyclic?
d) What is the minimum number of edges in G if G is connected graph and contain at least one cycle?
e) What is the minimum degree among all vertices in G if G is connected graph?
f) What is the maximum length of any simple path in G?

Solution 2

a) $\binom{|V|}{2} = \frac{|V|(|V|-1)}{2}$, as we have $\binom{|V|}{2}$ vertex pairs at maximum and each pair represents an edge.

b) $\binom{|V|-2}{2} = \frac{(|V|-2)(|V|-3)}{2}$. We only have $\binom{|V|-2}{2}$ vertex pairs at maximum since 2 vertices have degree 0 (i.e. cannot pair with other vertices).

c) $|V| - 1$, when G is a connected graph.

Solution 2

d) $|V|$. For a connected graph, the minimum number of edges is $|V| - 1$ (i.e. acyclic) and the graph will contain cycle when we add one more edge. Therefore, the minimum number of edges in G is $|V|$.

e) For $|V| < 2$, minimum degree is 0. For $|V| > 1$, minimum degree is 1, otherwise the graph will not be connected.

f) $|V| - 1$. The longest simple path will traverse all vertices in G exactly once and so the length of the path is $|V| - 1$.