COMP3711: Design and Analysis of Algorithms

Tutorial 3

HKUST

Question 1

Give asymptotic tight bounds for $T(n)$ by master theorem.

(a)

$$\begin{aligned} T(1) &= 1 \\ T(n) &= 3T(n/4) + n \qquad \text{if } n > 1 \end{aligned}$$

(b)

$$\begin{aligned} T(1) &= 1 \\ T(n) &= 3T(n/4) + 1 \qquad \text{if } n > 1 \end{aligned}$$

(c)

$$\begin{aligned} T(1) &= 1 \\ T(n) &= 4T(n/2) + n^2 \qquad \text{if } n > 1 \end{aligned}$$

(d)

$$\begin{aligned} T(1) &= 1 \\ T(n) &= 4T(n/3) + n^2 \qquad \text{if } n > 1 \end{aligned}$$

Question 2

Consider the HIRE-ASSISTANT algorithm in the lecture note, assuming that the candidates are presented in a random order, what is the probability that you hire exactly one time? What is the probability that you hire exactly n times?

Question 3

Use indicator random variables to solve the following problem, which is known as the **hat-check problem**. Each of n customers gives a hat to a hat-check person at a restaurant. The hat-check person gives the hats back to the customers in a random order. What is the expected number of customers who get back their own hat?

Question 4

Let $A[1..n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an **inversion** of A. Suppose that the elements of A form a uniform random permutation of $\langle 1, 2, ..., n \rangle$. Use indicator random variables to compute the expected number of inversions.