PA4

Coded by Zhao Zixuan

Generating test files

These tests generates symmetric (numbers with similar lengths) test cases.

Multiplication

```
Export["1.txt", Prepend[#, Length[#]] &@
  Flatten[Table[{x, y, x y} /. {x → RandomInteger[{10^#1, 10^(#1 + 1)}],
      y → RandomInteger[{10^(#1), 10^(#1 + 1)}]}, 100] & /@ Range[1, 100, 2], 1]]
```

Division

```
Export["1.txt", Prepend[#, Length[#]] &@
  Flatten[Table[{x, y, Floor[x / y]} /. {x → RandomInteger[{10^#1, 10^(#1 + 1)}],
      y → RandomInteger[{10^#1, 10^(#1 + 1)}]}, 20] & /@ Range[1, 100, 2], 1]]
```

Other tests are generated using similar code.

Benchmark and Algorithm Choosing

Multiplication

In[572]:=
```
fft = Mean /@ Partition[Flatten@Import["fft.txt", "Data"], 100];
```

In[573]:=
```
karatsuba = Mean /@ Partition[Flatten@Import["karatsuba.txt", "Data"], 100];
```

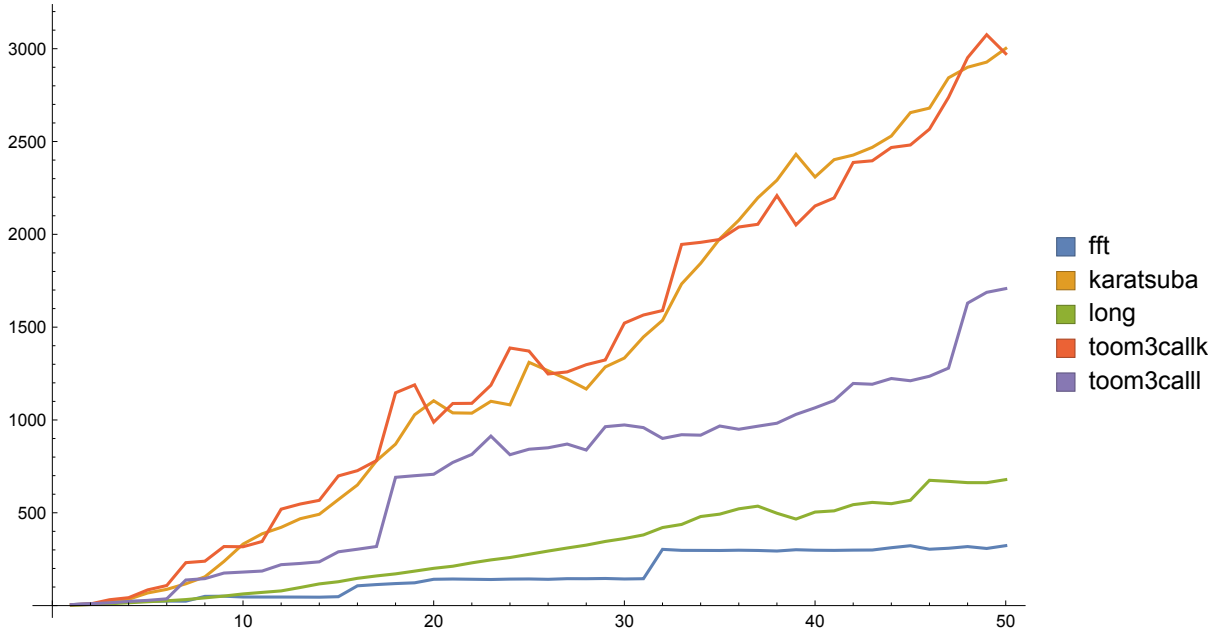
In[574]:=
```
long = Mean /@ Partition[Flatten@Import["long.txt", "Data"], 100];
```

In[576]:=
```
toom3calll = Mean /@ Partition[Flatten@Import["toom3_calll.txt", "Data"], 100];
```

In[577]:=
```
toom3callk = Mean /@ Partition[Flatten@Import["toom3_callk.txt", "Data"], 100];
```

In[601]:=
```
ListPlot[{fft, karatsuba, long, toom3callk, toom3calll}, PlotLegends →
  SwatchLegend[{{"fft", "karatsuba", "long", "toom3callk", "toom3calll"}}],
 Joined → True, ImageSize → Large]
```
绘制点集 绘图的图例 样本图例 连接点 真 图像尺寸 大

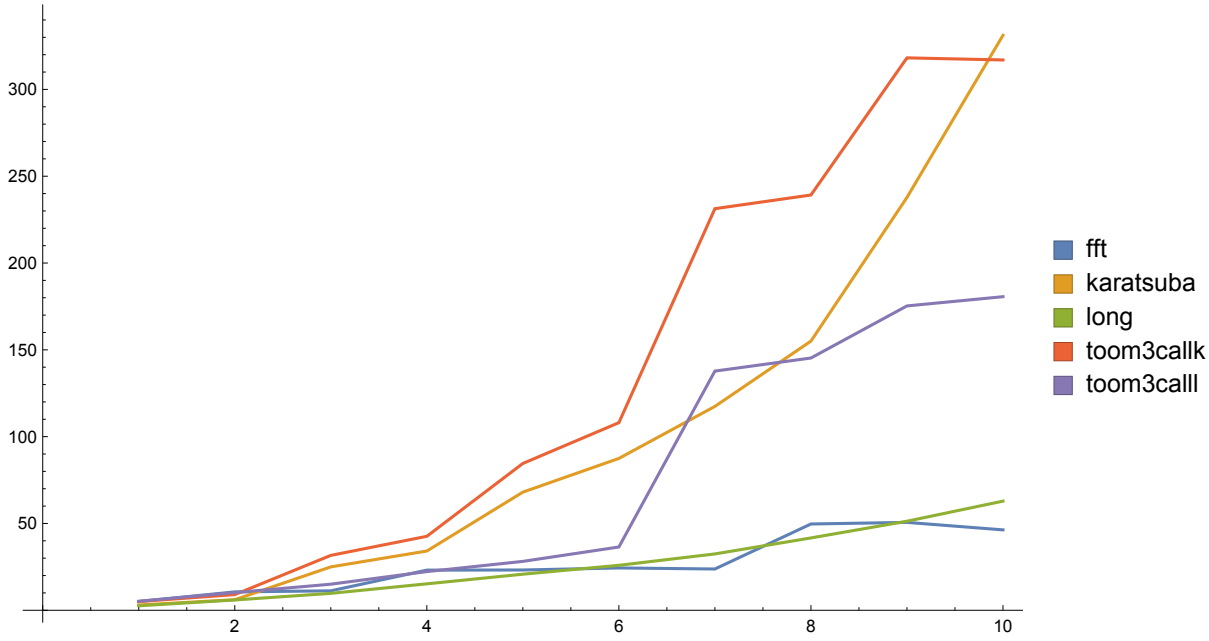
Out[601]=



The algorithm implemented are:
Long multiplication: $O(n^2)$
Karatsuba multiplication: $O(n^{1.585})$
Toom-Cook 3 Way multiplication: $O(n^{1.465})$
Fast Fourier Transform Multiplication: $O(n \log(n))$
However, FFT and long multiplication outproformed others(maybe due to optimization problems).

In[602]:=
```
ListPlot[
 {fft, karatsuba, long, toom3callk, toom3calll}[[All, 1 ;; 10]], PlotLegends →
  SwatchLegend[{{"fft", "karatsuba", "long", "toom3callk", "toom3calll"}}],
 Joined → True, ImageSize → Large]
```
绘制点集 全部 绘图的图例 样本图例 连接点 真 图像尺寸 大

Out[602]=



Hence for operand less than 10^{12}, long multiplication is used, and fft is used for larger multiplications.

Division

In[608]:=
```
longdiv = Mean /@ Partition[Flatten@Import["long.txt", "Data"], 20];
```
平均值 划分 压平 导入

In[609]:=
```
newtondiv = Mean /@ Partition[Flatten@Import["newton.txt", "Data"], 20];
```
平均值 划分 压平 导入

In[616]:=
```
ListPlot[{longdiv, newtondiv}, PlotLegends → SwatchLegend[{{"long", "newton"}}],
 Joined → True, ImageSize → Large]
```
绘制点集 绘图的图例 样本图例 连接点 真 图像尺寸 大

Out[616]=

In[611]:=
```
datalong = Transpose@{Range[50], longdiv};
```
转置 范围

In[612]:=
```
datanewton = Transpose@{Range[50], newtondiv};
```
转置 范围

In[613]:=
```
lmlong = LinearModelFit[datalong, x, x]
```
线性拟合模型

Out[613]=
```
FittedModel[-18.3665 + 5.58673 x]
```

In[614]:=
```
lmnewton = LinearModelFit[datanewton, x, x]
```
线性拟合模型

Out[614]=
```
FittedModel[13.7592 + 4.57007 x]
```

In[615]:=
```
Solve[Normal@lmlong == Normal@lmnewton, x]
```
解方程 转换为普通表达式 转换为普通表达式

Out[615]=
```
{{x → 31.5994}}
```

Hence long division is used for operands less than 10^{62}, and Newton division is used for larger numbers.