PA3 Matrix Class Verification Test

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1. Matrix Creation

1.1 Real Matrix

In[1]:=
```
(mat1 = {{2, 4, 0}, {3, 5, 5}}) // MatrixForm
                                   [矩阵格式
```

Out[1]//MatrixForm=

$$\begin{pmatrix} 2 & 4 & 0 \\ 3 & 5 & 5 \end{pmatrix}$$

1.2 Complex Matrix

In[2]:=
```
(mat2 = {{2, 4 + 2 I, 3 I}, {3 + I, 5, 5 + 0.35 I}, {2, 0, 1}}) // MatrixForm
                 [⋯    [虚数单位 [虚数单位        [虚数单位          [矩阵格式
```

Out[2]//MatrixForm=

$$\begin{pmatrix} 2 & 4 + 2 i & 3 i \\ 3 + i & 5 & 5. + 0.35 i \\ 2 & 0 & 1 \end{pmatrix}$$

2. Element-wise Operations

2.1 Element-wise Plus

In[3]:=
```
(mat3 = mat1 + mat1) // MatrixForm
                        [矩阵格式
```

Out[3]//MatrixForm=

$$\begin{pmatrix} 4 & 8 & 0 \\ 6 & 10 & 10 \end{pmatrix}$$

2.2 Element-wise Times

In[4]:=
```
(mat4 = mat1 * mat3) // MatrixForm
                        [矩阵格式
```

Out[4]//MatrixForm=

$$\begin{pmatrix} 8 & 32 & 0 \\ 18 & 50 & 50 \end{pmatrix}$$

3. Basic Matrix Calculations

3.1.1 Dot Product

In[5]:=
```
Dot[mat1, {{2, 5}, {5, 3}, {6, 4}}] // MatrixForm
```
点积 矩阵格式

Out[5]//MatrixForm=

$$\begin{pmatrix} 24 & 22 \\ 61 & 50 \end{pmatrix}$$

3.1.2 Complex Dot Product

In[6]:=
```
Dot[mat2, {{2, 5 + I}, {5, 3}, {6 + 3 I, 4 I}}] // MatrixForm
```
点积 虚数单位 … 虚数单位 矩阵格式

Out[6]//MatrixForm=

$$\begin{pmatrix} 15. + 28.\,i & 10. + 8.\,i \\ 59.95 + 19.1\,i & 27.6 + 28.\,i \\ 10. + 3.\,i & 10. + 6.\,i \end{pmatrix}$$

3.2.1 Inverse

In[7]:=
```
Inverse[{{4, 2, 6}, {3, 5, 2}, {1, 3, 1.5}}] // Chop // MatrixForm
```
逆 近似到零 矩阵格式

Out[7]//MatrixForm=

$$\begin{pmatrix} 0.06 & 0.6 & -1.04 \\ -0.1 & 0 & 0.4 \\ 0.16 & -0.4 & 0.56 \end{pmatrix}$$

3.2.2 Complex Inverse

In[8]:=
```
Inverse[{{4 + I, 2, 6}, {3, 5 - I, 2}, {I, 3, 1.5}}] // MatrixForm
```
逆 虚数单位 虚数单位 虚数单位 矩阵格式

Out[8]//MatrixForm=

$$\begin{pmatrix} 0.0373484 - 0.00776071\,i & 0.225546 + 0.147939\,i & -0.450121 - 0.166209\,i \\ -0.0873888 - 0.0143088\,i & 0.1346 - 0.00848828\,i & 0.170089 + 0.068553\,i \\ 0.169604 + 0.00371867\,i & -0.170574 - 0.133387\,i & 0.215683 + 0.162975\,i \end{pmatrix}$$

4. Basic Matrix Operations

4.1 Transpose

In[9]:=
```
Transpose[mat1] // MatrixForm
```
转置 矩阵格式

Out[9]//MatrixForm=

$$\begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 0 & 5 \end{pmatrix}$$

4.2 Conjugate Transpose

In[10]:=
```
ConjugateTranspose[mat2] // MatrixForm
```
共轭转置 矩阵格式

Out[10]//MatrixForm=

$$\begin{pmatrix} 2 & 3-i & 2 \\ 4-2i & 5 & 0 \\ -3i & 5.-0.35i & 1 \end{pmatrix}$$

4.3 Trace

In[11]:=
```
Tr[mat2]
```
迹

Out[11]= 8

4.4 Determinant

In[12]:=
```
Det[mat2]
```
行列式

Out[12]= 38.6 - 17.2 i

4.5 Permanent

In[13]:=
```
Permanent[mat2]
```
积和式

Out[13]= 58.6 + 62.8 i

---

5. Matrix Functions

5.1 Matrix Power

In[14]:=
```
MatrixPower[{{1, 2}, {3, 4}}, 5] // MatrixForm
```
矩阵的幂 矩阵格式

Out[14]//MatrixForm=

$$\begin{pmatrix} 1069 & 1558 \\ 2337 & 3406 \end{pmatrix}$$

In[15]:=
```
MatrixPower[mat2, 10] // MatrixForm
```
矩阵的幂 矩阵格式

Out[15]//MatrixForm=

$$\begin{pmatrix} -2.63777\times10^8+3.47648\times10^8\,i & -3.30257\times10^8+4.6941\times10^8\,i & -3.42453\times10^8+2.8606\times \\ -1.8244\times10^8+5.4098\times10^8\,i & -2.14088\times10^8+7.19801\times10^8\,i & -3.05684\times10^8+4.97513 \\ -4.9648\times10^7+1.11122\times10^8\,i & -5.99193\times10^7+1.48407\times10^8\,i & -7.48703\times10^7+9.94361 \end{pmatrix}$$

6. Linear Systems

6.1 Solve a Linear System

In[16]:=
```
Solve[x1 + 2 x2 + 3 x3 + 4 x4 == 4 &&
 2 x1 + 3 x2 + 4 x3 + 5 x4 == 3 && 3 x1 + 4 x2 + 5 x3 + 6 x4 == 2, {x1, x2}]
```
解方程

Out[16]=
```
{{x1 → -6 + x3 + 2 x4, x2 → 5 - 2 x3 - 3 x4}}
```

6.2 Null Space (Kernel)

In[17]:=
```
NullSpace[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}] // Transpose // MatrixForm
```
零空间 转置 矩阵格式

Out[17]//MatrixForm=

$$\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

6.3 Range

In[18]:=
```
MatrixRange[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
```

Out[18]=
```
MatrixRange[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
```

6.4 Rank

In[19]:=
```
MatrixRank[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
```
矩阵的秩

Out[19]=
```
2
```

6.5.1 RowReduce

In[20]:=
```
RowReduce[mat2] // Chop // MatrixForm
```
行约化 近似到零 矩阵格式

Out[20]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[21]:=
```
RowReduce[{{1, 2, 3, 4}, {2, 3, 4, 5}, {3, 4, 5, 6}}] // MatrixForm
```
行约化 矩阵格式

Out[21]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

6.5.2 RowReduce with Step-by-step Solution

In[22]:=
```
RowReduce[{{1, 2, 3, 4}, {2, 3, 4, 5}, {3, 4, 5, 6}}, ShowSteps → True]
```
行约化 真

RowReduce::optx : Unknown option ShowSteps in RowReduce[{{1, 2, 3, 4}, {2, 3, 4, 5}, {3, 4, 5, 6}}, ShowSteps → True]. >>

Out[22]=
```
RowReduce[{{1, 2, 3, 4}, {2, 3, 4, 5}, {3, 4, 5, 6}}, ShowSteps → True]
```

7. Advanced Matrix Operations

7.1 Eigenvalues

In[23]:=
```
mat61 = {{5, 2, 5, 6, 3}, {4, 3.5, 10, 6, 9},
    {1.7, 1, 5.5, 3, 2}, {6, -2, 5, 4, 6}, {1, 4, 3.2, 1, 0}};
```

In[24]:=
```
Eigenvalues[mat61] // Chop // MatrixForm
```
特征值 近似到零 矩阵格式

Out[24]//MatrixForm=

$$\begin{pmatrix} 16.7697 \\ 4.24049 \\ -2.6932 + 1.01439\,i \\ -2.6932 - 1.01439\,i \\ 2.3762 \end{pmatrix}$$

In[25]:=
```
mat62 = {{1 + I, 2, 3}, {3 I, 2, 5}, {4 + 2.5 I, 3 I, 7 + 5 I}};
```
虚数单位 虚数单位 … 虚数单位 虚数单

In[26]:=
```
Eigenvalues[mat62] // MatrixForm
```
特征值 矩阵格式

Out[26]//MatrixForm=

$$\begin{pmatrix} 9.73596 + 6.27851\,i \\ -0.897398 - 1.79526\,i \\ 1.16144 + 1.51674\,i \end{pmatrix}$$

7.2 Eigenvetors

In[27]:=
```
# / Last[#] & /@ Eigenvectors[mat61] // Chop // Transpose // MatrixForm
```
最后一个 特征向量 近似到零 转置 矩阵格式

Out[27]//MatrixForm=

$$\begin{pmatrix} 1.80177 & -0.46725 & 1.02779 + 1.04594\,i & 1.02779 - 1.04594\,i & 0.436349 \\ 2.59057 & 1.46498 & -0.682847 + 0.0765369\,i & -0.682847 - 0.0765369\,i & 1.41003 \\ 1.02879 & -0.094224 & 0.328499 + 0.361335\,i & 0.328499 - 0.361335\,i & -1.07301 \\ 1.31353 & -0.85066 & -2.04079 - 1.49396\,i & -2.04079 + 1.49396\,i & -0.266653 \\ 1. & 1. & 1. & 1. & 1. \end{pmatrix}$$

In[28]:=
```
# / Last[#] & /@ Eigenvectors[mat62] // Chop // Transpose // MatrixForm
```
最后一个 特征向量 近似到零 转置 矩阵格式

Out[28]//MatrixForm=

$$\begin{pmatrix} 0.306951 - 0.252406\,i & -0.950403 - 0.320301\,i & -0.895157 + 1.07644\,i \\ 0.506919 - 0.29238\,i & -1.04601 + 1.63218\,i & -1.85038 - 0.144394\,i \\ 1. & 1. & 1. \end{pmatrix}$$